F.6 Mathematics 2019 Mock Exam Paper I

Joe Cheung & his Partners

1. $\dfrac{(-ab)^{-4}}{a^{-2}b^{6}}$

$= \dfrac{a^{-4}b^{-4}}{a^{-2}b^{6}}$ 1M

$= a^{-4-(-2)}b^{-4-6}$

$= a^{-2}b^{-10}$ 1M

$= \dfrac{1}{a^2b^{10}}$ 1A

2. $\dfrac{a+2}{a} = \dfrac{5-b}{b}$

$b(a+2) = a(5-b)$

$ab + 2b = 5a - ab$

$2ab + 2b = 5a$ 1M

$2b(a+1) = 5a$ 1M

$b = \dfrac{5a}{2(a+1)}$ 1A

3. $\dfrac{a-3}{a+1} + \dfrac{a-5}{2-a}$

$= \dfrac{(a-3)(2-a)+(a-5)(a+1)}{(a+1)(2-a)}$ 1M

$= \dfrac{2a-a^2-6+3a+a^2+a-5a-5}{(a+1)(2-a)}$ 1M

$= \dfrac{a-11}{(a+1)(2-a)}$ / $\dfrac{11-a}{(a+1)(a-2)}$ 1A

4. (a) $m^2 + 12m + 36 = (m+6)^2$ 1A

(b) $n^2 - m^2 - 12m - 36$

$= n^2 - (m^2 + 12m + 36)$ 1M

$= n^2 - (m+6)^2$ 1M

$= (n + m + 6)(n - m - 6)$ 1A

5. *English Version*

Number of employees in the company 1A

$=\dfrac{186}{60\%}$

$=310$

Number of female employees which are married in the company 1A

$=310\times 30\%-31$

$=62$

The required percentage 1M

$=\dfrac{62}{310\times 40\%}\times 100\%$ 1A

$=50\%$

5. *中文版本*

該公司的員工數目 1A

$=\dfrac{186}{60\%}$

$=310$

該公司的已婚女性員工數目 1A

$=310\times 30\%-31$

$=62$

所求百分數 1M

$=\dfrac{62}{310}\times 100\%$ 1A

$=20\%$

6. (a) $3x < x+6$ or $2(x-1) < 6$

$2x < 6$ or $x-1 < 3$

$x < 3$ or $x < 4$ 1M + 1M

$\therefore\ x < 4$ 1A

(b) 1, 2, 3 1A

7. (a) $r = 5 + 13 = 18$ or $r = 13 - 5 = 8$ 1A

$\theta = 70^\circ$ or $\theta = 70^\circ + 180^\circ = 250^\circ$ 1A

(b) $\angle POS = 160^\circ - 70^\circ = 90^\circ$ 1M

$\therefore \quad OS \perp PQ$ 1A

8. (a) $z = k_1 + \dfrac{k_2}{xy}$ where k_1 and k_2 are non-zero constants. 1A

其中 k_1 及 k_2 為非零常數。

$2 = k_1 + \dfrac{k_2}{(6)(7)}$

$2 = k_1 + \dfrac{k_2}{42}$ ......(1)

$5 = k_1 + \dfrac{k_2}{(2)(3)}$

$5 = k_1 + \dfrac{k_2}{6}$ ......(2)

(2) – (1),

$3 = (\dfrac{1}{6} - \dfrac{1}{42})k_2$

$k_2 = 21$

Put $k_2 = 21$ into (2),

$5 = k_1 + \dfrac{21}{6}$

$k_1 = \dfrac{3}{2}$ 1M

$\therefore \quad z = \dfrac{3}{2} + \dfrac{21}{xy}$ 1A

(b) $z = \dfrac{3}{2} + \dfrac{21}{xy}$

$-2 = \dfrac{3}{2} + \dfrac{21}{x(2)}$ 1M

$-\dfrac{7}{2} = \dfrac{21}{2x}$

$x = -3$ 1A

9. Median 中位數 $= \dfrac{53+50+x+}{2} = \dfrac{103+x}{2}$ 1M

$\begin{cases} 5 \le x \le 9 \\ 5 \le y \le 9 \\ 3 \le x \le 9 \\ 0 \le y \le 6 \\ x \le y \end{cases} \Rightarrow \begin{cases} 5 \le x \le 6 \\ 5 \le y \le 6 \\ x \le y \end{cases}$ 1M

For median to be an integer, *x* must be odd.

中位數是整數，*x* 必為奇數。

$\because \quad 5 \le x \le 6$

$\therefore \quad x = 5$ 1A

For $x = 5$, $y = 6$,

Number of modes 眾數數量 = 3

$\therefore$ Number of modes of the above data will not increase after Peter resigns.

若當彼得離職後，以上數據的眾數數量不會增加。

For $x = 5$, $y = 5$,

Number of modes 眾數數量 = 2 1A

$\therefore$ Number of modes of the above data will increase after Peter resigns.

若當彼得離職後，以上數據的眾數數量增加。

$\therefore$ The age of Peter 彼得年齡 = 45 1A

10. (a) (i) Let $P(x, y)$.

$$(x-0)^2+(y-7)^2=(y-3)^2$$ 1M

$$x^2+y^2-14y+49=y^2-6y+9$$

$$x^2+40=8y$$

$$y=\frac{1}{8}x^2+5$$ 1A

(ii) L is a parabola.

L 為一條拋物線。 1A

(b) Vertex of L = (0, 5)

Equation of C:

$$(x-0)^2+(y-7)^2=2^2$$

$$x^2+y^2-14y+49=4$$

$$x^2+y^2-14y+45=0$$ 1A

$$\begin{cases} y=\frac{1}{8}x^2+5 & \text{........(1)} \\ x^2+y^2-14y+45=0 & \text{........(2)} \end{cases}$$

Sub. (1) into (2),

$$x^2+(\frac{1}{8}x^2+5)^2-14(\frac{1}{8}x^2+5)+45=0$$

$$x^2+\frac{1}{64}x^4+\frac{5}{4}x^2+25-\frac{7}{4}x^2-70+45=0$$

$$\frac{1}{64}x^4+\frac{1}{2}x^2=0$$

$$x^2(x^2+32)=0$$

$$x=0 \quad \text{or} \quad x^2=-32 \text{ (rejected)(捨去)}$$

1M

$\therefore$ The circle intersect L at only one point.

該圓形與 L 只有一個相交點。 1A

11. (a) Let :

the height of the outer cylinder 外圓柱體的高= H

the radius of the outer cylinder 外圓柱體的半徑= R

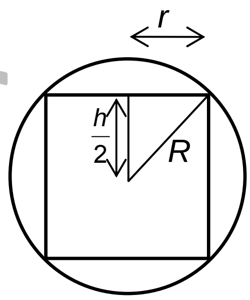
the height of the inner cylinder 內圓柱體的高= h

the radius of the inner cylinder 內圓柱體的半徑= r

$\therefore \quad \frac{H}{R}=\frac{h}{r}=\frac{1}{x}$

Consider the vertical section of the figure,

$$(\frac{h}{2})^2+r^2=R^2 \qquad \text{1M}$$



Side view 側視圖

$$\frac{h^2}{4}+(hx)^2=(Hx)^2$$

$$\frac{h^2}{4}+h^2x^2=(2r)^2x^2$$

$$\frac{h^2}{4}+h^2x^2=4r^2x^2$$

$$\frac{h^2}{4}+h^2x^2=4(hx)^2x^2$$

$$\frac{1}{4}+x^2=4x^4$$

$$1+4x^2=16x^4$$

$$16x^4-4x^2-1=0$$

$$(2x)^4-(2x)^2-1=0 \qquad \text{1A}$$

11. (b) From (a),

$$(2x)^4-(2x)^2-1=0$$
$$4(2x)^4-4(2x)^2-4=0$$
$$[2(2x)^2-1-\sqrt{5}][2(2x)^2-1+\sqrt{5}]=0$$
$$2(2x)^2=1+\sqrt{5} \quad \text{or} \quad 2(2x)^2=1-\sqrt{5}$$
$$(2x)^2=\frac{1+\sqrt{5}}{2} \quad \text{or} \quad (2x)^2=\frac{1-\sqrt{5}}{2} \text{ (rej.)}$$
$$x=\frac{\sqrt{1+\sqrt{5}}}{2\sqrt{2}}$$ 1A

$\therefore \quad \frac{R}{r}=\frac{H}{h}=2x$

$$\frac{R}{r}=2\times\frac{\sqrt{1+\sqrt{5}}}{2\sqrt{2}}=\frac{\sqrt{1+\sqrt{5}}}{\sqrt{2}}$$ 1A

$\therefore$ The radius of the outer cylinder : the radius of the inner cylinder

外圓柱體的半徑 ： 內圓柱體的半徑

$=\sqrt{\sqrt{5}+1}:\sqrt{2}$.

(c) Ratio 比 $=\frac{\pi R^2H}{\pi r^2h}=\frac{R^3}{r^3}$ 1A

$$=\frac{(\sqrt{\sqrt{5}+1})^3}{(\sqrt{2})^3}$$
$$=\frac{(\sqrt{5}+1)\sqrt{\sqrt{5}+1}}{2\sqrt{2}}$$
$$=\frac{(\sqrt{5}+1)\sqrt{\sqrt{5}+1}}{2\sqrt{2}}\cdot\frac{\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}-1}}$$
$$=\frac{\sqrt{5}+1}{\sqrt{2(\sqrt{5}-1)}}$$

$\therefore$ The volume of the outer cylinder : the volume of the inner cylinder

外圓柱體的體積 ： 內圓柱體的體積

$=\sqrt{5}+1:\sqrt{2(\sqrt{5}-1)}$. 1A

12. (a) maximum absolute error 最大絕對誤差= 0.5 kg 1A

(b) 64.5 kg ≤ weight of Peter <u>彼得</u>體重 < 65.5 kg 1A + 1A

(c) 175 cm ≤ height of Peter <u>彼得</u>身高 < 185 cm

1.75 m ≤ height of Peter <u>彼得</u>身高 < 1.85 m

$\frac{64.5}{1.85^2} < \text{BMI} < \frac{65.5}{1.75^2}$

18.8 < BMI < 21.4 1A + 1A

(d) Let *w* and *h* be the exact weight and height of Peter,

設 *w* 及 *h* 分別為<u>彼得</u>體重及身高的真確值。

Under the new measurements assume *w* = 65kg ; *h* = 180cm ;

根據新的量度工具，假設 *w* = 65kg ; *h* = 180cm

$\text{BMI} = \frac{65}{1.8^2} = 20.0617284$

Note that 64.75 ≤ *w* < 65.25 ; 177.5 ≤ *h* <182.5

∴ 19.44079565 < BMI < 20.71017655

Under original measurement,

根據原來的量度工具，

Max Absolute Error for BMI

BMI 的最大絕對誤差

= $21.3877551 - 20.0617284 = 1.3260267$

Under new measurement,

根據新的量度工具，

Max Absolute Error for BMI

BMI 的最大絕對誤差

= 20.71017655 – 20.0617284 = 0.64844815 1A

∴ Disagreed.

不同意。 1A

13. (a) $\angle BAP = \angle BPA = \angle QPC = \angle PQC = 45^{o}$

$CD = AB = BP = 2$

$PC = CQ = x - 2$

$0 < CQ \le CD$ 1M

$0 < x - 2 \le 2$

$2 < x \le 4$ 1A

(b) $RD = DQ = 2 - (x - 2) = 4 - x$

$RQ = \sqrt{(4-x)^2 + (4-x)^2} = \sqrt{2}(4-x)$

$PQ = \sqrt{(x-2)^2 + (x-2)^2} = \sqrt{2}(x-2)$ } 1A

Area 面積

$= RQ \times PQ$ 1M

$= \sqrt{2}(4-x) \cdot \sqrt{2}(x-2)$

$= 2(4-x)(x-2)$

$= 2(4x - 8 - x^2 + 2x)$

$= 8x - 16 - 2x^2 + 4x$

$= -2x^2 + 12x - 16$ 1A

(c) $\frac{AB}{SP}=\frac{BC}{SR}$

$\frac{2}{\sqrt{2}(4-x)}=\frac{x}{\sqrt{2}(x-2)}$ 1M

$2(x-2)=x(4-x)$

$x^2-2x-4=0$

$x=\frac{2\pm2\sqrt{5}}{2}=1\pm\sqrt{5}$

$x=1+\sqrt{5}$ or $x=1-\sqrt{5}$ (rejected)(捨去) 1A

Area 面積

$=-2(1+\sqrt{5})^2+12(1+\sqrt{5})-16$

$=-12-4\sqrt{5}+12+12\sqrt{5}-16$

$=8\sqrt{5}-16$ 1A

14. (a) $f(x) = (x + a)(x + b)(x + c) - 3$

$f(0) = 5$

$(0 + a)(0 + b)(0 + c) - 3 = 5$

$abc = 8$

$abc - 8 = 0$ 1A

(b) $a = 1$, $b = 2$, $c = 4$ or $a = -2$, $b = -1$, $c = 4$ 1A + 1A

(c) (i) For $a = 1$, $b = 2$, $c = 4$,

$f(x) = (x + 1)(x + 2)(x + 4) - 3$

$f(x) = x^3 + 3x^2 + 2x + 4x^2 + 12x + 8 - 3$

$f(x) = x^3 + 7x^2 + 14x + 5$

$g(x) = -x^2(x + 6) - 8x = -x^3 - 6x^2 - 8x$ 1A

$\therefore$ $f(x) + g(x)$

$= x^3 + 7x^2 + 14x + 5 - x^3 - 6x^2 - 8x$

$= x^2 + 6x + 5$ 1A

For $a = -2$, $b = -1$, $c = 4$,

$f(x) = (x - 2)(x - 1)(x + 4) - 3$

$f(x) = x^3 + x^2 - 10x + 5$

$\therefore$ $f(x) + g(x)$

$= x^3 + x^2 - 10x + 5 - x^3 - 6x^2 - 8x$

$= -5x^2 - 18x + 5$ 1A

(ii) For $a = 1$, $b = 2$, $c = 4$,

$$f(x+2) = -g(x+2)$$

$$f(x+2) + g(x+2) = 0$$

$$(x+2)^2 + 6(x+2) + 5 = 0$$

$$(x+2+1)(x+2+5) = 0$$

$$(x+3)(x+7) = 0$$

$$x = -3 \quad \text{or} \quad x = -7$$ 1A

For $a = -2$, $b = -1$, $c = 4$,

$$f(x+2) = -g(x+2)$$

$$f(x+2) + g(x+2) = 0$$

$$-5(x+2)^2 - 18(x+2) + 5 = 0$$

$$5(x+2)^2 + 18(x+2) - 5 = 0$$

$$x + 2 = \frac{-18 \pm \sqrt{18^2 - 4(5)(-5)}}{2(5)} = \frac{-9 \pm \sqrt{106}}{5}$$

$$x = \frac{-19 \pm \sqrt{106}}{5}$$ 1A

15. $\dfrac{\log_{\frac{1}{2}} x}{-5} + \dfrac{\log_4 y}{\frac{5}{2}} = 1$ 1M

$\dfrac{\log_{\frac{1}{2}} x}{-5} + \dfrac{2\log_4 y}{5} = 1$

$-\log_{\frac{1}{2}} x + 2\log_4 y = 5$

$-\log_{\frac{1}{2}} x + \log_4 y^2 = 5\log_4 4$

$\log_4 y^2 - \dfrac{\log_4 x}{\log_4 \frac{1}{2}} = \log_4 4^5$ 1M

$\log_4 y^2 - \dfrac{\log_4 x}{-\frac{1}{2}} = \log_4 4^5$

$\log_4 y^2 + \log_4 x^2 = \log_4 4^5$ 1M

$\log_4 x^2 y^2 = \log_4 4^5$

$x^2 y^2 = 1024$

$xy = 32$

$y = 32x^{-1}$

$\therefore \quad a = 32,\ b = -1$ 1A

16. (a) The required probability 所求概率

$= \dfrac{C_2^6 \times C_4^4}{2^6 - 2}$ 1M

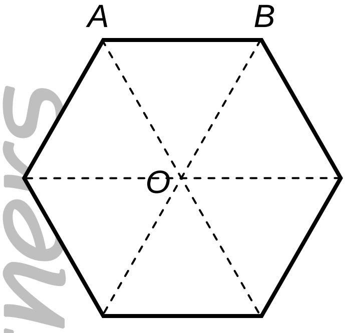
$= \dfrac{15}{62}$ 1A

(b) The required probability 所求概率

$= \dfrac{15}{62} \times \dfrac{15}{62}$ 1M

$= \dfrac{225}{3844}$ 1A

17. (a) $\angle AOB = \dfrac{360^\circ}{6} = 60^\circ$



Area 面積 $= 6 \times \dfrac{1}{2}(a)(a)\sin 60^\circ$ 1M

$= 6 \times \dfrac{1}{2}(a)(a)(\dfrac{\sqrt{3}}{2})$

$= \dfrac{3}{2}\sqrt{3}\,a^2 \text{ cm}^2$ 1A

(b) (i) The shaded area of the regions between the 1st and the 2nd hexagons

第 1 個正六邊形與第 2 個正六邊形之間的著色部分的面積

$= 3 \times \dfrac{1}{2}(\dfrac{a}{2})(\dfrac{a}{2})\sin 120^\circ = 3 \times \dfrac{1}{2}(\dfrac{a}{2})(\dfrac{a}{2})(\dfrac{\sqrt{3}}{2})$ 1M

$= \dfrac{3}{16}\sqrt{3}\,a^2 \text{ cm}^2$ 1A

(ii) The shaded area of the regions between the 2nd and the 3rd hexagons

第 2 個正六邊形與第 3 個正六邊形之間的著色部分的面積

$= 3 \times \dfrac{1}{2}(\dfrac{\sqrt{3}}{4}a)(\dfrac{\sqrt{3}}{4}a)\sin 120^\circ$

$= 3 \times \dfrac{1}{2}(\dfrac{\sqrt{3}}{4}a)(\dfrac{\sqrt{3}}{4}a)(\dfrac{\sqrt{3}}{2})$

$= \dfrac{9}{64}\sqrt{3}\,a^2 \text{ cm}^2$ 1M

The total shaded area 總著色部分的面積 $= \dfrac{42591\sqrt{3}}{65536}a^2$

$$\frac{\frac{3}{16}\sqrt{3}\,a^2[1-(\frac{3}{4})^n]}{1-\frac{3}{4}} = \frac{42591\sqrt{3}}{65536}a^2$$ 1M

$$1-(\frac{3}{4})^n = \frac{14197}{16384}$$

$$(\frac{3}{4})^n = \frac{2187}{16384}$$

$$n = 7$$

$\therefore$ the number of hexagons that Peter cut = 8 1A

彼得共剪出的六邊形數量= 8

18. (a) $AC = AE = 4$; $\angle ACE = \angle AEC = 50^\circ$

$CE = 2\times 4\cos 50^\circ = 5.142300877$

$(BC)^2 = (CE)^2 + (EB)^2 - 2(CE)(EB)\cos\angle BEC$

$= (5.142300877)^2 + (3)^2 - 2(5.142300877)(3)\cos 130^\circ$

$BC = 7.43476308$ 1A

$\cos\angle BCE = \dfrac{(CE)^2 + (BC)^2 - (EB)^2}{2(CE)(BC)}$ 1M

$= \dfrac{(5.142300877)^2 + (7.43476308)^2 - (3)^2}{2(5.142300877)(7.43476308)}$

$\angle BCE = 18.00539015^\circ$

$\approx 18.0^\circ$ 1A

(b) (i) $AA' = 4\sin 50^\circ = 3.064177772 \approx 3.06$ 1A

(ii) $A'E = 4\cos 50^\circ = 2.571150439$

In $\Delta EA'B$,

$(A'B)^2 = (A'E)^2 + (EB)^2 - 2(A'E)(EB)\cos\angle A'EB$

$= (2.571150439)^2 + (3)^2 - 2(2.571150439)(3)\cos 130^\circ$

$A'B = 5.052428747$ 1A

The required angle 所求交角= $\angle ABA'$

$\tan\angle ABA' = \dfrac{AA'}{A'B} = \dfrac{3.064177772}{5.052428747}$

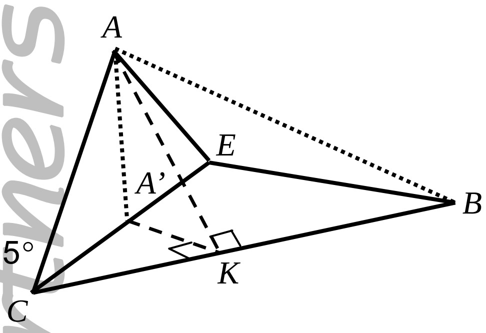
$\angle ABA' = 31.23581439^\circ \approx 31.2^\circ$ 1A

(iii) Draw a line from A to BC to meet at K such that $AK \perp BC$.

繪畫一條直線由 A 到 BC 相交於 K 使 $AK \perp BC$。

$\because$ $AK \perp BC$ and $AA' \perp CE$

$\therefore$ $A'K \perp BC$



$A'K = A'C \sin\angle BCE$

$= 2.571150439 \sin 18.00539015°$

$= 0.794759221$ 1A

The required angle 所求交角 $= \angle AKA'$

$\tan\angle AKA' = \dfrac{AA'}{A'K} = \dfrac{3.064177772}{0.794759221}$ 1M

$\angle AKA' \approx 75.5°$ 1A

19. (a) (i) $\angle BCG = 180^o - \angle CBE$ (int. ∠s, *BE* // *CG*)(同旁內角，*BE* // *CG*) 1A

$= 180^o - (180^o - \angle CDG)$ (opp. ∠s, cyclic quad.)(圓內接四邊形對角) 1A

$= \angle CDG$

(ii) $\angle CBH = \angle EBH$ (equal arcs, equal ∠s)(等弧對等角) 1A

$= \angle CHB$ (alt. ∠s, *BE* // *CG*)(錯角，*BE* // *CG*) 1A

$\therefore$ $BC = HC$ (sides opp. equal ∠s)(等角對邊相等)

(b) (i) $G(-\frac{144}{25}, -6)$

$\therefore$ x-coordinates of $H = -\frac{144}{25}$

Let $BH : DH = m : n$

$$-\frac{144}{25} = \frac{m(-8) + n(0)}{m+n}$$ 1M

$$144(m+n) = 200m$$

$$144m + 144n = 200m$$

$$144n = 56m$$

$$\frac{m}{n} = \frac{18}{7}$$

$\therefore$ $BH : DH = 18 : 7$ 1A

(ii) Let $H(-\frac{144}{25}, k)$

$$k = \frac{18(-6) + 7(0)}{18+7}$$ 1M

$$= -\frac{108}{25}$$

$\therefore$ $H(-\frac{144}{25}, -\frac{108}{25})$ 1A

(c) Slope of $DC = \dfrac{-6-\dfrac{42}{25}}{-8+\dfrac{144}{25}} = \dfrac{24}{7}$

Slope of $BC = \dfrac{0-\dfrac{42}{25}}{0+\dfrac{144}{25}} = -\dfrac{7}{24}$ 1M

Slope of DC × Slope of BC $= \dfrac{24}{7} \times -\dfrac{7}{24} = -1$ 1A

∴ The claim is correct.

該宣稱正確。

∴ Agreed.

同意。 1A

F.6 Mathematics 2019 Mock Exam Paper II

Joe Cheung & his Partners

Mock MC Solution

1.	B	16.	C	31.	C
2.	A	17.	B	32.	D
3.	C	18.	C	33.	C
4.	B	19.	D	34.	A
5.	C	20.	C	35.	A
6.	A	21.	A	36.	B
7.	C	22.	B	37.	C
8.	C	23.	A	38.	A
9.	A	24.	C	39.	A
10.	C	25.	B	40.	A
11.	D	26.	C	41.	C
12.	C	27.	D	42.	B
13.	B	28.	C	43.	C
14.	B	29.	D	44.	A
15.	C	30.	B	45.	C

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1. B

$$\frac{(x^{n-1})^2}{x^{2n-1}} = \frac{x^{2n-2}}{x^{2n-1}} = x^{2n-2-2n+1} = x^{-1} = \frac{1}{x}$$

2. A

$$\frac{u+mv}{w} = w + \frac{mu}{v}$$

$$v(u+mv) = w^2v + muw$$

$$vu + mv^2 = w^2v + muw$$

$$vu - muw = w^2v - mv^2$$

$$u(v-mw) = v(w^2 - mv)$$

$$u = \frac{v(w^2-mv)}{v-mw}$$

3. C

$$\alpha^2 - 4\alpha - 9\beta^2 - 12\beta$$

$$= \alpha^2 - 9\beta^2 - 4\alpha - 12\beta$$

$$= (\alpha+3\beta)(\alpha-3\beta) - 4(\alpha+3\beta)$$

$$= (\alpha+3\beta)(\alpha-3\beta-4)$$

4. B

$$\frac{1}{5x-3} - \frac{1}{5x+3}$$

$$= \frac{(5x+3)-(5x-3)}{(5x-3)(5x+3)}$$

$$= \frac{5x+3-5x+3}{25x^2-9}$$

$$= \frac{6}{25x^2-9}$$

5. C

✓ I. *b* is a root of the equation f(*x*) = 0.

b 為方程 f(*x*) = 0 的根。

✗ II. *b* must lie between *a* and *c*.

b 必定在 *a* 及 *c* 之間。

✓ III. f(*x*) = 0 has two distinct real roots.

f(*x*) = 0 有兩個相異實根。

6. A

y-intercept of $L_1 = -\frac{b}{2} < 0 \Rightarrow b > 0$

x-intercept of $L_1 = -\frac{b}{a} < 0 \Rightarrow a > 0$

x-intercept of $L_2 = -d > 0 \Rightarrow d < 0$

y-intercept of $L_2 = -\frac{d}{c} > 0 \Rightarrow c > 0$

✓ I. $bd < 2$

✓ II. $ad < b$

✗ III. $bc < 2d$

7. C

$$\begin{aligned}&f(x)-f(x-1)\\&=\frac{1}{2}x(x+1)-\frac{1}{2}(x-1)(x-1+1)\\&=\frac{1}{2}x(x+1)-\frac{1}{2}x(x-1)\\&=\frac{1}{2}x(x+1-x+1)\\&=\frac{1}{2}x(2)\\&=x\end{aligned}$$

8. C

$$8(\frac{1}{2})^3-a(\frac{1}{2})+b=0$$

$$1-\frac{1}{2}a+b=0$$

$$b=\frac{1}{2}a-1$$

Remainder 餘數 $=8(-\frac{1}{2})^3-a(-\frac{1}{2})+b$

$$\begin{aligned}&=-1+\frac{1}{2}a+b\\&=-1+\frac{1}{2}a+\frac{1}{2}a-1\\&=a-2\end{aligned}$$

9. A

Let $\$x$ be the cost of the book.

設$\$x$ 為這本書的成本。

$$480\times(1-20\%)=x(1+20\%)$$
$$384=1.2x$$
$$x=320$$

$\therefore$ The cost of the book is \$320.

這本書的成本為\$320。

10. C

$a:b=b:c=c:d=k:1$

$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\frac{k}{1}$

$\therefore$ $c=dk$

$b=ck \Rightarrow b=dk^2$

$a=bk \Rightarrow a=dk^3 \Rightarrow \frac{a}{d}=k^3$

$\therefore$ $a:d=k^3:1$

11. D

$v=\frac{k\sqrt{w}}{x^2}$ where k is a constant 其中 k 為常數。

$\frac{vx^2}{\sqrt{w}}=k$

$\frac{v^2x^4}{w}=k^2$

$\frac{w}{v^2x^4}=\frac{1}{k^2}$

$\frac{w}{x^4v^2}=$ constant 常數

12. C

$a_1 = 1$

$a_2 = 3a_1 = 3$

$a_3 = 5a_2 = 15$

$a_4 = 7a_3 = 105$

$a_5 = 9a_4 = 945$

$a_6 = 11a_5 = 10395$

$a_7 = 13a_6 = 135135$

13. B

$$6 \times 3 = (6 + x) \times 2$$
$$18 = 12 + 2x$$
$$6 = 2x$$
$$x = 3$$

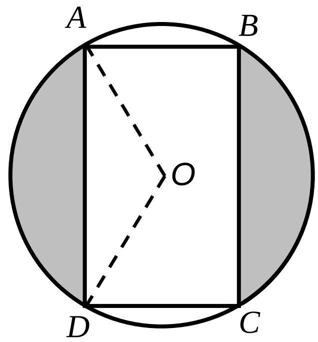
14. B

Lower limit of the area 面積的下限 $= 39.5 \times 19.5 - 12.5 \times 32.5 = 364\ \text{m}^2$

Upper limit of the area 面積的上限 $= 40.5 \times 20.5 - 11.5 \times 31.5 = 468\ \text{m}^2$

$\therefore \quad 364 < x < 468$

15. C



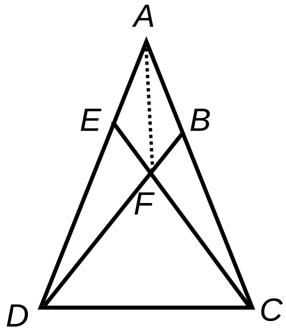
$$\tan\frac{\angle AOD}{2}=\frac{3\sqrt{3}}{3}$$

$$\angle AOD=120^\circ$$

$$OA=\sqrt{3^2+(3\sqrt{3})^2}=6$$

Area 面積 $=[\pi(6)^2\times\frac{120^\circ}{360^\circ}-\frac{6\sqrt{3}\times 3}{2}]\times 2=44\text{ cm}^2$

16. C



Join AF,

Let the area of $\Delta DEF = A$ and area of $\Delta BCF = B$

Area of $\Delta AEF=\frac{A}{3}$; Area of $\Delta ABF=\frac{B}{2}$

Area of $\Delta ABD=\frac{B}{2}+\frac{A}{3}+A=\frac{4A}{3}+\frac{B}{2}$

Area of $\Delta AEC=\frac{A}{3}+\frac{B}{2}+B=\frac{A}{3}+\frac{3B}{2}$

Area of $\Delta DFC=2\times(\frac{4A}{3}+\frac{B}{2})-B=\frac{8A}{3}$

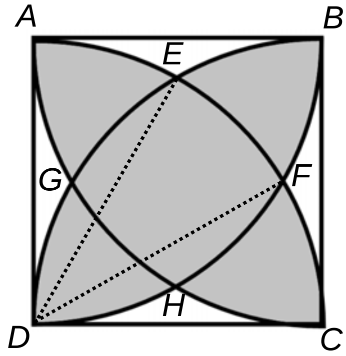
Area of $\Delta DFC=3\times(\frac{A}{3}+\frac{3B}{2})-A=\frac{9B}{2}$

$\therefore\quad \frac{8A}{3}=\frac{9B}{2}$

$\frac{A}{B}=\frac{27}{16}$

$\therefore$ Area of ΔDEF : Area of ΔBCF = 27 : 16

17. B



$\angle ADE = \angle EDF = \angle FDC = 30^{o}$

$\therefore \quad \overset{\frown}{AE} = \overset{\frown}{EF} = \overset{\frown}{FC}$

Similarly, $\overset{\frown}{AG} = \overset{\frown}{GH} = \overset{\frown}{HC}$, $\overset{\frown}{DG} = \overset{\frown}{GE} = \overset{\frown}{EB}$, $\overset{\frown}{BF} = \overset{\frown}{FH} = \overset{\frown}{HD}$

$\therefore \quad \overset{\frown}{AE} = \overset{\frown}{EF} = \overset{\frown}{FC} = \overset{\frown}{AG} = \overset{\frown}{GH} = \overset{\frown}{HC} = \overset{\frown}{DG} = \overset{\frown}{GE} = \overset{\frown}{EB} = \overset{\frown}{BF} = \overset{\frown}{FH} = \overset{\frown}{HD}$

$\overset{\frown}{AC} = 2\pi(5) \times \frac{90^\circ}{360^\circ} = \frac{5\pi}{2}$

$\therefore \quad \overset{\frown}{AE} = \overset{\frown}{EF} = \overset{\frown}{FC} = \overset{\frown}{AG} = \overset{\frown}{GH} = \overset{\frown}{HC} = \overset{\frown}{DG} = \overset{\frown}{GE} = \overset{\frown}{EB} = \overset{\frown}{BF} = \overset{\frown}{FH} = \overset{\frown}{HD} = \frac{5\pi}{6}$

$\therefore$ Perimeter

$= 8 \times \frac{5\pi}{6}$

$= 20.9$

18. C

$180^{o} - \angle A - \angle B + 180^{o} - \angle C - \angle D + 180^{o} - \angle E - \angle F + 180^{o} - \angle G - \angle H + 180^{o} - \angle I - \angle J = 180^{o}$

$-(\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H + \angle I + \angle J) = -720^{o}$

$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H + \angle I + \angle J = 720^{o}$

19. D

$\angle AED = \angle CED$ (given)(已知)

$ED = ED$ (common side)(公共邊)

$AE = CE$ (corr. sides, ≅Δs)(全等Δ對應邊)

$\Delta ADE \cong \Delta CDE$ (SAS)

$\therefore \angle ADE = \angle CDE = \dfrac{180^\circ}{2} = 90^\circ$

$\therefore BD \perp AC$

$\therefore$ II ✓

$AD = CD$ (corr. sides, ≅Δs)(全等Δ對應邊)

$\therefore$ III ✓

$\angle BAE = \angle BCE$ (given)(已知)

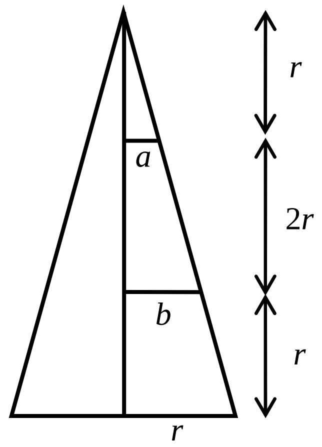
$\angle AEB = \angle CEB$ (given)(已知)

$BE = BE$ (common side)(公共邊)

$\Delta ABE \cong \Delta CBE$ (AAS)

$\therefore$ I ✓

20. C



$\frac{a}{r} = \frac{r}{r+2r+r} \Rightarrow a = \frac{1}{4}r$

$\frac{b}{r} = \frac{r+2r}{r+2r+r} \Rightarrow b = \frac{3}{4}r$

The overlapping volume 重疊的體積

$= \frac{1}{3}\pi b^2(3r) - \frac{1}{3}\pi a^2(r)$

$= \frac{1}{3}\pi(\frac{3}{4}r)^2(3r) - \frac{1}{3}\pi(\frac{1}{4}r)^2(r)$

$= \frac{9}{16}\pi r^3 - \frac{1}{48}\pi r^3$

$= \frac{13}{24}\pi r^3$

21. A

Let $BF = AG = AH = a$

$\angle EFG = \angle AHG = \angle AGH = \angle EGF = \frac{180^\circ - 100^\circ}{2} = 40^\circ$

$\angle BEF = 80^\circ$

$\tan\angle BEF = \frac{BF}{BE} \Rightarrow BE = \frac{a}{\tan 80^\circ}$

$\tan\angle EGF = \frac{BF}{BG} \Rightarrow BG = \frac{a}{\tan 40^\circ}$

$GE = BG - BE = \frac{a}{\tan 40^\circ} - \frac{a}{\tan 80^\circ}$

$AE = AG + GE = a + \frac{a}{\tan 40^\circ} - \frac{a}{\tan 80^\circ}$

$AD = AB = AG + BG = a + \frac{a}{\tan 40^\circ}$

$\tan\angle ADE = \frac{AE}{AD} = \frac{a + \frac{a}{\tan 40^\circ} - \frac{a}{\tan 80^\circ}}{a + \frac{a}{\tan 40^\circ}} = \frac{1 + \frac{1}{\tan 40^\circ} - \frac{1}{\tan 80^\circ}}{1 + \frac{1}{\tan 40^\circ}}$

$\angle ADE = 43^\circ$

22. B

Join OC,

$\overset{\frown}{AB} : \overset{\frown}{CD} = 3 : 2$

$\therefore \quad \angle AOB : \angle COD = 3 : 2$

Let $\angle AOB = 3a$ and $\angle COD = 2a$.

$\angle CAD = \dfrac{\angle COD}{2} = a$

$$\angle CAD + \angle AOB + 60^\circ = 180^\circ$$
$$a + 3a = 120^\circ$$
$$a = 30^\circ$$

$\therefore \quad \angle CAD = 30^\circ$

$\angle ACD = 90^\circ$

$$\angle CDA + \angle ACD + \angle CAD = 180^\circ$$
$$\angle CDA + 90^\circ + 30^\circ = 180^\circ$$
$$\angle CDA = 60^\circ$$

23. A

I ✓

II ✓

III ✗

24. C

$(4, 330^\circ) \Rightarrow (2\sqrt{3}, -2) \Rightarrow (-2\sqrt{3}, -2)$

25. B

Let $A(a, b)$, $B(a, 0)$

$b = 2 - a^2$

$\therefore \quad A(a, 2 - a^2)$

Let $M(x, y)$,

$\therefore \quad x = a$; $y = \dfrac{2 - a^2}{2}$

$y = \dfrac{2 - x^2}{2} = 1 - \dfrac{x^2}{2}$

26. C

$(x + 2t)^2 + (y + t)^2 = 9$

Centre 圓心 = $(-2t, -t)$; Radius 半徑 = 3

The shortest distance 最短距離

$= \sqrt{(-3 + 2t)^2 + (1 + t)^2} - 3$

$= \sqrt{4t^2 - 12t + 9 + 1 + 2t + t^2} - 3$

$= \sqrt{5t^2 - 10t + 10} - 3$

$\sqrt{5t^2 - 10t + 10} - 3 = 2$

$\sqrt{5t^2 - 10t + 10} = 5$

$5t^2 - 10t + 10 = 25$

$t^2 - 2t - 3 = 0$

$t = 3$ or $t = -1$

27. D

$2x^2 + 2y^2 + 8x - 6y - 3 = 0 \Rightarrow x^2 + y^2 + 4x - 3y - \frac{3}{2} = 0$

$A(-\frac{4}{2}, -\frac{-3}{2}) = A(-2, \frac{3}{2})$

$\therefore$ I ✗

Sub. (1, 2),

$(1)^2 + (2)^2 + 4(1) - 3(2) - \frac{3}{2} = \frac{3}{2} > 0$

$\therefore$ II ✓

$OA = \sqrt{(-2-0)^2 + (\frac{3}{2} - 0)^2} = \frac{5}{2}$

$OB = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}$

$AB = \sqrt{(-2-1)^2 + (\frac{3}{2} - 2)^2} = \frac{\sqrt{37}}{2}$

$\cos \angle AOB = \frac{(\frac{5}{2})^2 + (\sqrt{5})^2 - (\frac{\sqrt{37}}{2})^2}{2(\frac{5}{2})(\sqrt{5})}$

$\angle AOB = 80^\circ$

$\therefore$ III ✓

28. C

All possible outcomes 所有結果:

(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (2, 1), (3, 1), (4, 1), (5, 1), (3, 2), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)

∴ Number of all possible outcomes = 20

所有結果的數量= 20

Favourable outcomes 合適結果:

(1, 3), (1, 4), (2, 4), (2, 5), (3, 5), (3, 1), (4, 1), (4, 2), (5, 2), (5, 3)

∴ Number of all favourable outcomes = 10

合適結果的數量= 10

The required probability 所求概率

$$=\frac{10}{20}=\frac{1}{2}$$

29. D

The mean age of 60 teachers in a school ten years ago = 30

十年前 60 名教師的平均年齡為 = 30

The mean age of 60 teachers in a school now = 40

現在 60 名教師的平均年齡為 = 40

The mean age of 50 teachers in a school now

現在 50 名教師的平均年齡為

$$=\frac{60\times 40-10\times 40}{50}=40$$

30. B

$\frac{b+c}{2}=15 \Rightarrow b+c=30$

$\frac{c+d}{2}=23 \Rightarrow c+d=46$

$\frac{b+d}{2}=21 \Rightarrow b+d=42$

$\frac{a+e}{2}=23 \Rightarrow a+e=46$

$$c+d+b+d-(b+c)=46+42-30$$
$$2d=58$$
$$d=29$$

$$a+b+c+d+e=a+e+b+d+c+d-d=46+42+46-29=105$$

mean of $\{a, b, c, d, e\}=\frac{a+b+c+d+e}{5}=\frac{105}{5}=21$

31. C

$$(1-i)^{4m-1}=\frac{(-2i)^{2m}}{1-i}=\frac{4^m}{1-i}=\frac{4^m}{1-i}\times\frac{1+i}{1+i}=\frac{4^m(1+i)}{2}=\frac{4^m}{2}(1+i)=2^{2m-1}(1+i)$$

32. D

$h(x) = 3 \log_3 x + 2$

$= \log_3 x^3 + \log_3 9$

$= \log_3 9x^3$

33. C

$$(a^2)^x = b^{\frac{1}{x}}$$

$$a^{2x} = b^{\frac{1}{x}}$$

$$\log a^{2x} = \log b^{\frac{1}{x}}$$

$$2x\log a = \frac{1}{x}\log b$$

$$2x^2\log a = \log b$$

$$2x^2 = \frac{\log b}{\log a}$$

$$x^2 = \frac{1}{2}\log_a b$$

34. A

$$\begin{aligned}18\times 4^{16}+5\times 16^3-16^3 &= (16+2)\times 16^8+4\times 16^3\\ &= 16^9+2\times 16^8+4\times 16^3\\ &= 1\times 16^9+2\times 16^8+4\times 16^3\\ &= 1200004000_{16}\end{aligned}$$

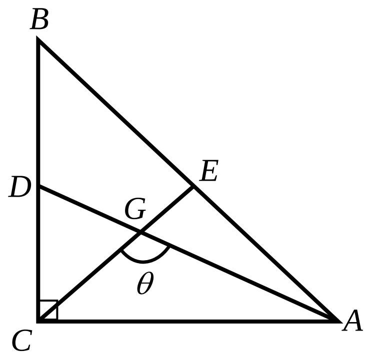
35. A

I. $h \geq 0$ ✓

II. $3h + k \leq 10$ ✓

III. $2h - k \leq -5$ ✗

36. B



Let $AC = BC = a$

$AB = \sqrt{a^2 + a^2} = \sqrt{2}a$; $AE = EB = \frac{\sqrt{2}}{2}a$

$\angle AEC = 90^\circ$

$CE = \sqrt{a^2 - (\frac{\sqrt{2}}{2}a)^2} = \frac{\sqrt{2}}{2}a$

$CG = \frac{\sqrt{2}}{2}a \times \frac{2}{3} = \frac{\sqrt{2}}{3}a$

$AD = \sqrt{a^2 + (\frac{1}{2}a)^2} = \frac{\sqrt{5}}{2}a$

$AG = \frac{\sqrt{5}}{2}a \times \frac{2}{3} = \frac{\sqrt{5}}{3}a$

$$\cos\theta = \frac{(\frac{\sqrt{2}}{3}a)^2 + (\frac{\sqrt{5}}{3}a)^2 - a^2}{2(\frac{\sqrt{2}}{3}a)(\frac{\sqrt{5}}{3}a)} = \frac{-1}{\sqrt{10}}$$

$$\sin\theta = \sqrt{1 - (\frac{-1}{\sqrt{10}})^2} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

37. C

$$\begin{cases} 3\alpha^2 - 2\alpha + 6 = 0 \\ 3\beta^2 - 2\beta + 6 = 0 \end{cases}$$

$\alpha + \beta = \frac{2}{3}$; $\alpha\beta = 2$

$$\begin{aligned} 4\alpha + 2\alpha\beta^2 &= 4\alpha + 2(\alpha\beta)\beta \\ &= 4\alpha + 4\beta \\ &= 4(\frac{2}{3}) \\ &= \frac{8}{3} \end{aligned}$$

38. A

Let S(n) be the sum of first n terms and d be the common difference.

設 S(n)為該數列的首 n 項之和及 d 為公差。

$$S(n)=\frac{n}{2}[2(95)+(n-1)d]$$

For n = 8, $S(8)=\frac{8}{2}[2(95)+(8-1)d]=760+28d$

For n = 9, $S(9)=\frac{9}{2}[2(95)+(9-1)d]=855+36d$

For n = 10, $S(10)=\frac{10}{2}[2(95)+(10-1)d]=950+45d$

$S(8)<S(9)$ and $S(9)>S(10)$

$760+28d<855+36d$ and $855+36d>950+45d$

$8d>-95$ and $9d<-95$

$d>-\frac{95}{8}$ and $d<-\frac{95}{9}$

$\therefore\quad -\frac{95}{8}<d<-\frac{95}{9}$

$\therefore\quad d=-11$

39. A

$$\frac{18\sin^2 x - 24\sin x + 14}{18\sin^2 x - 12\sin x + 9\cos^2 x}$$

$$= \frac{2(9\sin^2 x - 12\sin x + 9) - 4}{9\sin^2 x - 12\sin x + 9}$$

$$= 2 - \frac{4}{9\sin^2 x - 12\sin x + 9}$$

$$= 2 - \frac{4}{9(\sin^2 x - \frac{4}{3}\sin x) + 9}$$

$$= 2 - \frac{4}{9(\sin^2 x - \frac{4}{3}\sin x + \frac{4}{9} - \frac{4}{9}) + 9}$$

$$= 2 - \frac{4}{9(\sin x - \frac{2}{3})^2 + 5}$$

$\therefore$ Minimum value 最小值 $= 2 - \frac{4}{5} = \frac{6}{5}$

40. A

Let O be the centre of circle ABC.

設 O 為圓形 ABC 的圓心。

$\angle AOC = 2 \times \angle ABC = 126^\circ$

$$AC^2 = 5^2 + 5^2 - 2(5)(5)\cos 126^\circ$$
$$AC = 8.910065242$$

$$CD = 2 \times AC \cos 63^\circ$$
$$= 2 \times 8.910065242 \times \cos 63^\circ$$
$$= 8.090169944$$
$$\approx 8.09 \text{ cm}$$

41. C

$x^2 + y^2 - px - 14y + 53 = 0$

$(3)^2 + (6)^2 - p(3) - 14(6) + 53 > 0$ and $(\frac{p}{2})^2 + 7^2 - 53 > 0$

$14 - 3p > 0$ and $\frac{p^2}{4} - 4 > 0$

$p < \frac{14}{3}$ and $p^2 - 16 > 0$

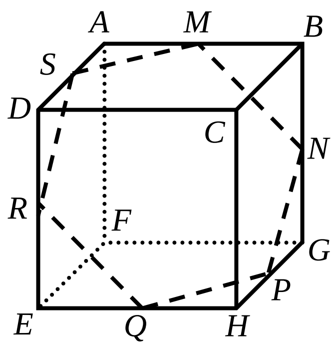
$p < \frac{14}{3}$ and $p < -4$ or $p > 4$

$\therefore$ $p < -4$ or $4 < p < \frac{14}{3}$

42. B

Let a cm be the side of the cube.

設 a cm 為該立方體的邊長。



$BF = \sqrt{a^2 + a^2} = \sqrt{2}a$ cm

$$\tan\theta = \frac{\frac{1}{2}a}{\frac{\sqrt{2}a}{4}} = \sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{1^2 + (\sqrt{2})^2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

43. C

$$2C_2^n - 2(n-1) + 3 = 33$$

$$2 \times \frac{n(n-1)}{2} - 2n = 28$$

$$n^2 - n - 2n - 28 = 0$$

$$n^2 - 3n - 28 = 0$$

$$(n-7)(n+4) = 0$$

$$n = 7 \quad \text{or} \quad n = -4 \text{ (rej.)}$$

44. A

The required probability

所求概率

$$= \frac{\frac{1}{6} \times \frac{4}{5} \times \frac{1}{4} + \frac{5}{6} \times \frac{1}{5} \times \frac{1}{4}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}}$$

$$= \frac{1}{4}$$

45. C

Variance 方差

$$= \left(\frac{12}{4}\right)^2$$

$$= 9$$