MOCK Paper 2020

Marking Scheme

Compiled by Joe

F.6 Mathematics 2020 Mock Exam Paper I

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Marking Scheme:

1. $9x-5=y(x+z)$

$9x-5=yx+yz$ 1M

$9x-yx=yz+5$

$x(9-y)=yz+5$ 1M

$x=\dfrac{yz+5}{9-y}$ 1A

2. $\dfrac{2}{x+7}-\dfrac{10}{x^2+12x+35}$

$=\dfrac{2}{x+7}-\dfrac{10}{(x+7)(x+5)}$ 1M

$=\dfrac{2(x+5)-10}{(x+7)(x+5)}$ 1M

$=\dfrac{2x+10-10}{(x+7)(x+5)}$

$=\dfrac{2x}{(x+7)(x+5)}$ 1A

3. $99^2+(33+r)^2=(62-3r)^2$ 1M

$9801+1089+66r+r^2=3844-372r+9r^2$

$8r^2-438r-7046=0$ 1M

$(4r-271)(r+13)=0$

$r=67.75$(rejected) or $r=-13$ 1A

4. (a) $6x^5 - 15x^2 + 60 - 24x^3$

$= 3x^2(2x^3 - 5) - 12(2x^3 - 5)$ 1M

$= (3x^2 - 12)(2x^3 - 5)$

$= 3(x^2 - 4)(2x^3 - 5)$

$= 3(x + 2)(x - 2)(2x^3 - 5)$ 1A

(b) $6x^5 - 15x^2 + 60 - 24x^3 + 6x^2y - 24y$.

$= 3(x + 2)(x - 2)(2x^3 - 5) + 6y(x^2 - 4)$

$= 3(x + 2)(x - 2)(2x^3 - 5) + 6y(x + 2)(x - 2)$ 1M

$= 3(x + 2)(x - 2)(2x^3 - 5 + 2y)$

$= 3(x + 2)(x - 2)(2x^3 + 2y - 5)$ 1A

5. (a) Cost price 成本

$= \dfrac{36000}{1 - 25\%}$ 1M

$= \$48000$ 1A

(b) The least selling price of the table

該餐桌的最低售價

$= 48000 \times (1 - 15\%)$ 1M

$= \$40800$

$> \$40000$

$\therefore$ The deal cannot be made. 1A

交易不能成功。

6. Let x and y be the present ages of Mary and her son respectively.

設 x 及 y 分別為小麗和她的兒子現在的年齡。

$$\begin{cases} x-3=8(y-3) & \text{......(1)} \\ x+2=4.5(y+2) & \text{......(2)} \end{cases}$$ 1M + 1M

From (1), $x=8(y-3)+3=8y-21$ ......(3)

Sub. (3) into (1),

$$\left.\begin{aligned} 8y-21+2&=4.5(y+2) \\ 8y-19&=4.5y+9 \\ 3.5y&=28 \\ y&=8 \end{aligned}\right\}$$ 1M

Put y = 8 into (3), $x=8(8)-21=43$

∴ The present ages of Mary and her son are 43 and 8 respectively. 1A

小麗和她的兒子現在的年齡分別為 43 及 8。

7. (a) P' = (2, –9) 1A

Q' = (–3, 6) 1A

(b) Slope of $PQ=\dfrac{-2-2}{-3-9}=\dfrac{1}{3}$; Slope of $P'Q'=\dfrac{-9-5}{2+3}=-3$ 1M

PQ 的斜率 $=\dfrac{-2-2}{-3-9}=\dfrac{1}{3}$; $P'Q'$ 的斜率 $=\dfrac{-9-5}{2+3}=-3$

∵ Slope of $PQ\times$ Slope of $P'Q'$ = –1

PQ 的斜率 $\times P'Q'$ 的斜率 = –1

∴ PQ is perpendicular to $P'Q'$. 1A

PQ 垂直於 $P'Q'$。

8. (a) $9+x\geq\frac{5x+3}{4}+8$

$1+x\geq\frac{5x+3}{4}$

$4+4x\geq 5x+3$ 1M

$-x\geq -1$

$x\leq 1$ 1A

(b) $9+x\geq\frac{5x+3}{4}+8$ and $3(x-2)<1-4x$

$x\leq 1$ and $3x-6<1-4x$ 1M

$x\leq 1$ and $7x<7$

$x\leq 1$ and $x<1$ 1A

$\therefore$ $x<1$

$\therefore$ the greatest integer satisfying both inequalities = 0 1A

滿足不等式的最大整數= 0

9. (a) The least possible total surface area

最小可能的總表面面積

$=(235+155)\times 2\times 45+235\times 155\times 2$ 1M

$=107950\text{ cm}^2$ 1A

(b) The least possible buckets

$=\frac{107950\div 20000\times 1000}{400+2.5}$ 1M

$=13.4099$ 1A

>13

$\therefore$ Disagree. 1A

不同意。

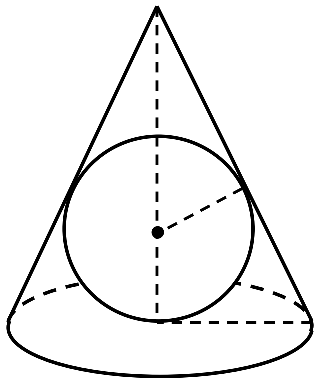
10. (a) Let the radius of the sphere, base radius of the cone and the height of the cone be r, R and H respectively.

設球體半徑、圓錐體底半徑及圓錐體高度分別為 r、R 及 H。

$$\frac{r}{R}=\frac{3}{7} \quad \Rightarrow \quad r=\frac{3R}{7}$$

$$\frac{H-r}{\sqrt{H^2+R^2}}=\frac{\sqrt{H^2+R^2}-R}{H}=\frac{r}{R}=\frac{3}{7}$$

$$\begin{cases} \dfrac{H-r}{\sqrt{H^2+R^2}}=\dfrac{3}{7} & \text{......(1)} \\ \dfrac{\sqrt{H^2+R^2}-R}{H}=\dfrac{3}{7} & \text{......(2)} \end{cases} \Rightarrow \begin{cases} \dfrac{H-\frac{3R}{7}}{\sqrt{H^2+R^2}}=\dfrac{3}{7} & \text{......(1)} \\ \dfrac{\sqrt{H^2+R^2}-R}{H}=\dfrac{3}{7} & \text{......(2)} \end{cases}$$ 1M



From (2), $\sqrt{H^2+R^2}=\frac{3}{7}H+R$ ……(3)

Sub. (3) into (1),

$$\frac{H-\frac{3R}{7}}{\frac{3}{7}H+R}=\frac{3}{7}$$

$$7(H-\frac{3R}{7})=3(\frac{3}{7}H+R)$$ 1M

$$\frac{40}{7}H=6R$$

$$\frac{H}{R}=\frac{21}{20}$$

∴ The required ratio = 21 : 20 1A

(b) Volume of the sphere 球體體積 $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi(\frac{3R}{7})^3=\frac{4}{3}\pi\frac{27R^3}{343}=\frac{36\pi R^3}{343}$ 1M

Volume of the cone 圓錐體體積 $=\frac{1}{3}\pi R^2 H$

∴ The required ratio $=\dfrac{\frac{36\pi R^3}{343}}{\frac{1}{3}\pi R^2 H}=\frac{108}{343}\cdot\frac{R}{H}=\frac{108}{343}\cdot\frac{20}{21}=\frac{720}{2401}$ = 720 : 2401 1A

11. (a) Median 中位數= 4.5

$8 + x = 9 + y + 4$

$x = y + 5$ 1M

$\because$ $4 < y < 8$ and $x > 10$

$4 + 5 < y + 5 < 8 + 5$ and $x > 10$

$9 < x < 13$ and $x > 10$

$\therefore$ $10 < x < 13$ 1A

For $x = 11, y = 6$; $x = 12, y = 7$ 1A

(b) (i) The median is the greatest when the scores of these two essays are both greater than 4.

當兩篇文章均高於 4 分時，中位數為最大。

$\therefore$ The greatest possible median of the scores is 5. 1A

分數的最大可取中位數為 5。

(ii) The mean is the least when the scores of these two essays are 3 and 4.

當兩篇文章分數為 3 和 4 時，平均數為最小。

By (a), there are two cases.

Case 1: $x = 11$ and $y = 6$

$$\text{Mean} = \frac{3\times9+4\times12+5\times9+6\times6+7\times4}{9+12+9+6+4} = 4.6$$ 1A

Case 2: $x = 12$ and $y = 7$

$$\text{Mean} = \frac{3\times9+4\times13+5\times9+6\times7+7\times4}{9+13+9+7+4} \approx 4.619$$ 1A

12. (a) $f(x) = x^3 + g(x)$

$f(5) = -3(5) + k$

$f(5) = k(5) - 27$

$\therefore\ -3(5) + k = k(5) - 27$ 1M

$-15 + k = 5k - 27$

$-4k = -12$

$k = 3$ 1A

(b) Let $f(x) = (x-5)^2(x+a) + 3x - 27$. 1M

$f(2) = -3(2) + 3$

$(2-5)^2(2+a) + 3(2) - 27 = -3$

$2 + a = 2$

$a = 0$

$\therefore\ f(x) = (x-5)^2(x) + 3x - 27$ 1M

$g(x) = f(x) - x^3$

$= x^3 - 10x^2 + 25x + 3x - 27 - x^3$

$= -10x^2 + 28x - 27$ 1A

(c) $g(x) = 0$

$-10x^2 + 28x - 27 = 0$

$10x^2 - 28x + 27 = 0$

$\Delta = (-28)^2 - 4(10)(27) = -296 < 0$ 1M

$\therefore$ $g(x) = 0$ has no real roots.

$g(x) = 0$ 沒有實根。

$\therefore$ Disagree. 1A

不同意。

13. (a) $\angle AFC = 180^{o} - \angle BCE = 180^{o} - y$ (int. $\angle$s, $BC//AD$)(同旁內角，$BC//AD$) 1M

$\angle AEC = 180^{o} - \angle ABC = 180^{o} - x$ (opp. $\angle$s, cyclic quad.)(圓內接四邊形對角)

$\angle AFC = \angle AEC + \angle DAE$ (ext. $\angle$ of Δ)(Δ外角) 1M

$180^{o} - y = 180^{o} - x + z$

$x = y + z$ 1A

(b) (i) Join AC.

$\angle BCA = \angle DCA = \dfrac{y}{2}$ (equal chords, equal $\angle$s)(等弦對等角)

$\angle BAD = 180^{o} - x$ (int. $\angle$s, $BC//AD$)(同旁內角，$BC//AD$)

$\angle BAC = \angle DAC = \dfrac{180^\circ - x}{2} = 90^\circ - \dfrac{x}{2}$

$\angle BAC + \angle BCA + \angle ABC = 180^{o}$ ($\angle$ sum of Δ)(Δ內角和)

$90^\circ - \dfrac{x}{2} + \dfrac{y}{2} + x = 180^\circ$

$\dfrac{y}{2} = 90^\circ - \dfrac{x}{2}$ 1A

$\therefore$ $\angle BCA = \angle BAC$

$\therefore$ $BA = BC \Rightarrow AE = CD$ 1A

$\angle EAF = \angle DCF$ ($\angle$s in the same segment)(同弓形內的圓周角) 1M

$\angle AEF = \angle CDF$ ($\angle$s in the same segment)(同弓形內的圓周角)

$\therefore$ $\Delta AFE \cong \Delta CFD$ (ASA)

(ii) From (b)(i),

$\dfrac{y}{2} = 90^\circ - \dfrac{x}{2} \Rightarrow x + y = 180^\circ$ 1M

$\therefore$ $AB//DF$ (int. $\angle$s supp.)(同旁內角互補) 1A

$\therefore$ $ABCF$ is a parallelogram.

$ABCF$ 為平行四邊形。

14. (a) $\Delta CBQ \sim \Delta CAP$ (AAA) 1M

$\frac{BQ}{AP} = \frac{CB}{CA} = \frac{5}{10}$

$\therefore \quad \frac{CB}{CA} = \frac{1}{2}$

$\therefore \quad CB : BA = 1 : 1$ 1M

Let $C(a, b)$.

$\frac{a+10}{2} = 1$; $\frac{b+3}{2} = 16$

$a = -8$ $b = 29$

$\therefore \quad C(-8, 29)$ 1A

(b) Let $P(h, k)$.

$AP = 10$

$AP = \sqrt{(10-h)^2 + (3-k)^2}$

$10^2 = (10-h)^2 + (3-k)^2$

$100 = 100 - 20h + h^2 + 9 - 6k + k^2$

$\therefore \quad h^2 + k^2 - 20h - 6k + 9 = 0$ ……(1) 1M

Slope of $AP \times$ Slope of $CP = -1$

$\frac{k-3}{h-10} \times \frac{k-29}{h+8} = -1$

$\frac{k^2 - 32k + 87}{h^2 - 2h - 80} = -1$

$k^2 - 32k + 87 = -h^2 + 2h + 80$

$\therefore \quad h^2 + k^2 - 2h - 32k + 7 = 0$ ……(2) 1M

By solving (1) and (2), $h = 16$, $k = 11$ or $h = 0.4$, $k = 0.2$ (rejected)

$\therefore \quad P(16, 11)$

$\therefore$ Slope of PQ = Slope of $CP = \frac{11-29}{16+8} = -\frac{3}{4}$ 1A

(c) Let the inclination of $PQ = \theta$

設 PQ 的傾角= θ

$\tan\theta = -\frac{3}{4}$ 1M

$\theta = \tan^{-1}(-\frac{3}{4})$

$\tan\angle PCA = \frac{10}{\sqrt{(16+8)^2+(11-29)^2}}$

$= \frac{1}{3}$

$\angle PCA = \tan^{-1}(\frac{1}{3})$

$\therefore$ The slope of another common tangent from C to C_1 and C_2

另一條由 C 到 C_1 及 C_2 的公切線斜率

$= -\tan[2\tan^{-1}(\frac{1}{3}) + \tan^{-1}(\frac{3}{4})]$

$= -\frac{24}{7}$ 1A

$\therefore$ The required equation

所求方程

$\frac{y-29}{x+8} = -\frac{24}{7}$

$7(y-29) = -24(x+8)$

$7y - 203 = -24x - 192$

$24x + 7y - 11 = 0$ 1A

15. $\frac{\log y - \log 81}{x - 0} = \frac{0 - \log 81}{4 - 0}$ 1M

$\frac{\log y - \log 81}{x} = \frac{-\log 81}{4}$

$4\log y - 4\log 81 = -x\log 81$

$4\log y = -x\log 81 + 4\log 81$

$\log y^4 = \log 81^{-x} + \log 81^4$

$\log y^4 = \log(81^4 \times 81^{-x})$ 1M

$y^4 = 81^{4-x}$

$y = 3^{4-x}$ 1A

16. (a) Number of arrangement 安排學生就坐的方法

$= 5! \times P_2^6$ 1M

$= 3600$ 1A

(b) Number of arrangement 安排學生就坐的方法

$= P_2^2 \times 3! \times P_2^4$ 1M

$= 144$ 1A

17. (a) $T_1 \bullet T_2 = 2 \bullet 6 = 12$

$T_2 \bullet T_3 = 6 \bullet 18 = 108$

$T_3 \bullet T_4 = 18 \bullet 54 = 972$

$\therefore$ $T_1 \bullet T_2 + T_2 \bullet T_3 + T_3 \bullet T_4 + \ldots + T_n \bullet T_{n+1}$

$= \dfrac{12(9^n - 1)}{9 - 1}$ 1M

$= \dfrac{3(9^n - 1)}{2}$ 1A

(b) Let the first term = a, common ratio = r

設首項= a，公比= r

$H_1 = T_1 \bullet T_2 \bullet T_3 \bullet \ldots \bullet T_n = a \bullet ar \bullet ar^2 \bullet \ldots \bullet ar^{n-1} = a^n \bullet r^{(0+1+2+\ldots+n-1)} = a^n \bullet r^{\frac{n(n-1)}{2}}$

$H_2 = T_2 \bullet T_3 \bullet T_4 \bullet \ldots \bullet T_{n+1} = ar \bullet ar^2 \bullet ar^3 \bullet \ldots \bullet ar^n = a^n \bullet r^{(1+2+\ldots+n)} = a^n \bullet r^{\frac{n(n+1)}{2}}$

$H_3 = T_3 \bullet T_4 \bullet T_5 \bullet \ldots \bullet T_{n+2} = ar^2 \bullet ar^3 \bullet ar^4 \bullet \ldots \bullet ar^{n+1} = a^n \bullet r^{(2+2+\ldots+n+1)} = a^n \bullet r^{\frac{n(n+3)}{2}}$

$\therefore$ $H_1, H_2, H_3, \ldots$ forms a geometric sequence. 1M

First term = $a^n \bullet r^{\frac{n(n-1)}{2}}$, common ratio = r^n 1A

$\therefore$ $T_1 \bullet T_2 \bullet T_3 \bullet \ldots \bullet T_n + T_2 \bullet T_3 \bullet T_4 \bullet \ldots \bullet T_{n+1} + \ldots + T_8 \bullet T_9 \bullet T_{10} \bullet \ldots \bullet T_{n+7}$

$= a^n \bullet r^{\frac{n(n-1)}{2}} + a^n \bullet r^{\frac{n(n+1)}{2}} + a^n \bullet r^{\frac{n(n+3)}{2}} + \ldots +$

$= \dfrac{a^n \bullet r^{\frac{n(n-1)}{2}} [(r^n)^8 - 1]}{r^n - 1}$

$= \dfrac{a^n \bullet r^{\frac{n(n-1)}{2}} (r^{8n} - 1)}{r^n - 1}$

$\dfrac{2^n \bullet 3^{\frac{n(n-1)}{2}} (3^{8n} - 1)}{3^n - 1}$ 1A

18. (a) $f(x) = 3x^2 - 6kx + 4k^2 - 9$

$f(x) = 3(x^2 - 2kx) + 4k^2 - 9$ 1M

$f(x) = 3(x^2 - 2kx + k^2 - k^2) + 4k^2 - 9$

$f(x) = 3(x - k)^2 - 3k^2 + 4k^2 - 9$

$f(x) = 3(x - k)^2 + k^2 - 9$ 1M

$\therefore$ $P(k, k^2 - 9)$ 1A

(b) $Q(k, -(k^2 - 9) + 6) = Q(k, 15 - k^2)$ 1A

(c) (i) $4k^2 - 9 = 55$ 1M

$4k^2 = 64$

$k^2 = 16$

$k = 4$ or $k = -4$(rejected) 1A

$\therefore$ $P(4, 7)$, $Q(4, -1)$, $R(0, 3)$

$\because$ Slope of $PR = \frac{7-3}{4-0} = 1$; Slope of $QR = \frac{-1-3}{4-0} = -1$

$\therefore$ $PR \perp QR$ 1M

$\therefore$ PQ is the dimeter of the required circle.

PQ 為所求圓形的直徑。

$\therefore$ Centre 圓心= (4, 3) 1A

(c) (ii) $\begin{cases} y = 3x^2 - 24x + 55 & \text{......(1)} \\ y = 7x - 25 & \text{......(2)} \end{cases}$

Sub. (2) into (1),

$$7x - 25 = 3x^2 - 24x + 55$$

$$3x^2 - 31x + 80 = 0$$

$$x = 5 \quad \text{or} \quad x = \frac{16}{3} \text{ (rejected)}$$

For $x = 5$, $y = 10$

$\therefore$ $H(5, 10)$ 1A

$\begin{cases} y = -3x^2 + 24x - 49 & \text{......(1)} \\ y = 7x - 25 & \text{......(2)} \end{cases}$

Sub. (2) into (1),

$$7x - 25 = -3x^2 + 24x - 49$$

$$3x^2 - 17x + 24 = 0$$

$$x = 3 \quad \text{or} \quad x = \frac{8}{3} \text{ (rejected)}$$

For $x = 3$, $y = -4$

$\therefore$ $K(3, -4)$

$\therefore$ Area of ΔPGH : Area of ΔQGK

ΔPGH 的面積 :ΔQGK 的面積

$= \dfrac{(7-3)\times(5-4)}{2} : \dfrac{[3-(-1)]\times(4-3)}{2}$ 1M

$= 1:1$ 1A

19. (a) (i) By cosine formula 利用餘式定理，

$4^2 + AB^2 - 2(4)(AB)\cos 36° = 5^2$ 1M

$AB^2 - 8\cos 36°(AB) - 9 = 0$

$AB = 7.648792$ or $AB = -1.176656$ (rejected)

$\therefore$ $AB = 7.65\text{ cm}$ 1A

(ii) By cosine formula 利用餘式定理，

$AC^2 = AB^2 + 3^2 - 2(3)(AB)\cos 36°$

$AC = 5.511443 = 5.51$ cm 1A

(iii) By cosine formula 利用餘式定理，

$DC^2 = AC^2 + 5^2 - 2(5)(AC)\cos 36°$

$DC = 3.284432$ 1A

By cosine formula 利用餘式定理，

$$\cos\angle CBD = \frac{3^2 + 4^2 - DC^2}{2(3)(4)}$$

$\angle CBD = 53.68759°$

$\therefore$ $\angle CBD = 53.7°$ 1A

(b) Let DH be the height of ΔBCD

設 DH 為ΔBCD 的高度。

$DH = 4\sin\angle CBD = 3.2232$ cm

$BH = 4\cos\angle CBD = 2.36875$ cm } 1A

Let K be the point on AB such that $HK \perp BC$.

設 K 為 AB 上的一點使 $HK \perp BC$。

$HK = BH\tan 36° = 1.720998$ cm

$BK = \dfrac{BH}{\cos 36°} = 2.927837$ cm

By cosine formula 利用餘式定理，

$DK^2 = BK^2 + 4^2 - 2(4)(BK)\cos 36°$

$DK = 2.371246$ cm 1A

$\therefore$ The inclination between the face BCD and ABC.

平面 BCD 及 ABC 之間的夾角

$= \cos^{-1}\dfrac{DH^2 + HK^2 - DK^2}{2(DH)(HK)} = 45.8466°$ 1M + 1A

$\therefore$ Height of the tetrahedron $ABCD$

四面體 $ABCD$ 的高度

$= DH\sin 45.8466° = 2.31527$ cm 1A

By Heron's formula, area of ΔABC = 6.74377 cm^2

利用希羅公式，ΔABC 的面積= 6.74377 cm^2 1A

$\therefore$ Volume of the tetrahedron $ABCD$

四面體 $ABCD$ 的體積

$= \dfrac{1}{3} \times 6.74377 \times 2.31527 = 5.1984898 \approx 5.20\ cm^3$ 1A

<u>*MC Solution:*</u>

1. D

$(4^{2n+2} \cdot 64)^2 = (2^{4n+4} \cdot 2^6)^2 = (2^{4n+10})^2 = 2^{8n+20}$

2. A

$$p(q+3) = r - 2q$$
$$pq + 3q = r - 2q$$
$$pq + 2q = r - 3q$$
$$q(p+2) = r - 3q$$
$$q = \frac{r-3q}{p+2}$$

3. C

$$3a^2x^2 - 3by + 9a^2y - bx^2$$
$$= 3a^2x^2 + 9a^2y - 3by - bx^2$$
$$= 3a^2(x^2+3y) - b(x^2+3y)$$
$$= (x^2+3y)(3a^2-b)$$

4. C

$$\frac{2}{x-3} + \frac{1}{6-2x}$$
$$= \frac{4}{2(x-3)} - \frac{1}{2(x-3)}$$
$$= \frac{3}{2(x-3)}$$

5. D

I. ∵ y – intercept < 0

∴ $a < 0$

II. ∵ $h = -\frac{b}{2a}$ ⇨ $b = -2ah$ ⇨ $b = -2(+)(-)$

∴ $b > 0$

III. ∵ The graph opens upwards.

圖像開口向上。

∴ $c > 0$

6. A

I. Consider L_1: $ax - 3y = b$

y – intercept 截距 $= -\frac{b}{3} > 0$ ⇨ $b < 0$

x – intercept 截距 $= \frac{b}{a} > 0$ ⇨ $a < 0$

Slope 斜率 $= \frac{a}{3}$

Consider L_2: $3x - cy = d$

x – intercept 截距 $= \frac{d}{3} < 0$ ⇨ $d < 0$

y – intercept 截距 $= -\frac{d}{c} > 0$ ⇨ $c > 0$

Slope 斜率 $= \frac{3}{c}$

∴ $abcd < 0$

II. ∵ y – intercept of L_1 = y – intercept of L_2

L_1 的 y –截距 = L_2 的 y –截距

∴ $-\frac{b}{3} = -\frac{d}{c}$ ⇨ $bc = 3d$

III. ∵ Slope of L_1 < Slope of L_2

L_1 的斜率 < L_2 的斜率

∴ $\frac{a}{3} < \frac{3}{c}$ ⇨ $ac < 9$

7. D

$f(x-2) = x^2 - 2x^3 + 3$

$f(2) = f(4-2)$

$= (4)^2 - 2(4)^3 + 3$

$= 16 - 128 + 3$

$= -109$

8. A

$x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

$= (x+1)(x^2 - 4x + 4)$

$\therefore \quad b = -4$

OR

$x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

$= (x-2)(x^2 - x - 2)$

$\therefore \quad b = -1$

$\therefore$ Sum of the possible values of $b = -4 + (-1) = -5$

有 b 的可能值之和 $= -4 + (-1) = -5$

9. B

Interest 利息

$= 60000 \times (1 + \frac{3.6\%}{6})^6 - 60000$

$= \$2\ 193$

10. D

$4.8 \times 250000 \div 100 \div 1000 \div 3$

$= 4$ cm

11. D

$b \propto a^2$ and $a \propto \dfrac{1}{c}$

$\therefore \quad a^2 \propto \dfrac{1}{c^2} \Rightarrow \quad b \propto a^2 \propto \dfrac{1}{c^2}$

$\therefore \quad \dfrac{b}{a^2} = \text{constant}$ 常數 ； $bc^2 = \text{constant}$ 常數 ； $a^2c^2 = \text{constant}$ 常數 $\Rightarrow ac = \text{constant}$ 常數

12. C

T(n) = T(n – 1) + T(n – 2)

T(1) = 1

T(2) = 3

T(3) = T(2) + T(1) = 3 + 1 = 4

T(4) = T(3) + T(2) = 4 + 3 = 7

T(5) = T(4) + T(3) = 7 + 4 = 11

T(6) = T(5) + T(4) = 11 + 7 = 18

T(7) = T(6) + T(5) = 18 + 11 = 29

T(8) = T(7) + T(6) = 29 + 18 = 47

$\therefore$ T(5) + T(8) = 47 + 11 = 58

13. A

$x^2 - bx + cx - bc \le 0$

$(x - b)(x + c) \le 0$

$\therefore \quad -c \le x \le b$

14. B

Maximum absolute error 最大絕對誤差

$= 5.5^3 - 5^3$

$= 41.375 \text{ cm}^3$

15. C

Let a and b be the length and the width of the small rectangle respectively.

設 a 及 b 分別為每個小長方形的長度和闊度。

$2(a + b) = 20$

$a + b = 10$ ......(1)

$5ab = (a + b)(2b)$ ......(2)

Sub. (1) into (2),

$5ab = 10(2b)$

$5ab = 20b$

$b(a - 4) = 0$

$b = 0$ (rejected) or $a = 4$

For $a = 4$, $b = 6$

$\therefore$ Total area of the five small rectangles

五個小長方形的總面積

$= 5(4)(6)$

$= 120$

16. C

$\Delta DEF \sim \Delta ABF \sim \Delta CEB$ (AAA)

$$\left(\frac{DE}{CE}\right)^2 = \frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta CEB}$$
$$\frac{9}{16} = \frac{6}{\text{Area of } \Delta CEB}$$
$$\text{Area of } \Delta CEB = \frac{32}{3}\text{ cm}^2$$

$$\left(\frac{DE}{AB}\right)^2 = \frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABF}$$
$$\frac{9}{49} = \frac{6}{\text{Area of } \Delta ABF}$$
$$\text{Area of } \Delta ABF = \frac{98}{3}\text{ cm}^2$$

$\because$ Height of ΔDEA = Height of ΔCEB

$\therefore$ Area of ΔDEA = $\frac{3}{4} \times \frac{32}{3} = 8\text{ cm}^2$

$\therefore$ Area of the quadrilateral $ABCE$

$= \text{Area of } \Delta ABF - \text{Area of } \Delta DEF - \text{Area of } \Delta DEA + \text{Area of } \Delta CEB$

$= \frac{98}{3} - 6 - 8 + \frac{32}{3}$

$= \frac{98}{3} - 6 - 8 + \frac{32}{3}$

$= 29\frac{1}{3}\text{ cm}^2$

17. C

Let r and O be the radius and the centre of the semi-circle $DEFG$.

設 r 及 O 分別為半圓 $DEFG$ 的半徑及圓心。

$$\frac{r}{6}=\frac{8-r}{8}$$
$$8r=48-6r$$
$$14r=48$$
$$r=\frac{24}{7}$$

$$\tan\angle GOC=\frac{8-\frac{24}{7}}{\frac{24}{7}}$$
$$\angle GOC=\tan^{-1}(\frac{4}{3})$$

∴ Area of the shaded region 陰影部分的面積

$$=\frac{1}{2}\times(8-\frac{24}{7})(\frac{24}{7})-\pi(\frac{24}{7})^2\times\frac{\tan^{-1}(\frac{4}{3})}{360}=2.39$$

18. B

Let P be the intersection of CQ and HI.

設 P 為 CQ 及 HI 的交點。

$$\angle DPI=\angle EDQ=\frac{360^\circ}{5}=72^\circ$$

$$\angle HIQ=\frac{360^\circ}{8}=45^\circ$$

∴ $z=72^\circ-45^\circ=27^\circ$

19. C

$\angle BPC = \angle APD = 110°$

$\angle ACB = 60°$

$\angle PBC = 180° - 110° - 60° = 10°$

$\therefore \quad \angle QDC = \angle PBC = 10°$

$\angle ACD = 180° - 10° - 10° - 60° = 100°$

$\angle CAD = \angle CDA = \dfrac{180° - 100°}{2} = 40°$

$\therefore \quad x = 40°$

20. B

$\angle CFD = \angle AFE = 180° - 120° = 60°$

$\angle EAD = 180° - 100° - 60° = 20°$

$\because \quad \angle CFD = \angle CDF = \angle DCF = 60°$

$\therefore \quad DF = CD = 3$ cm

By sine formula 利用正弦公式，

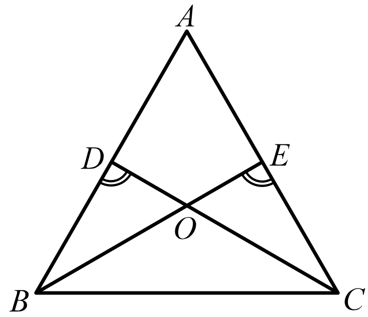
$$\frac{3}{\sin \angle FED} = \frac{5}{\sin 120°}$$
$$\angle FED = 31.30645°$$

$\angle ADE = 180° - 50° - 50° - 20° - 31.30645° = 28.69355°$

By sine formula 利用正弦公式，

$$\frac{AE}{\sin 28.69355°} = \frac{5}{\sin 20°}$$
$$AE = 7.02 \text{ cm}$$

21. B



I. $\because$ O is the circumcentre of ΔABC

O 為ΔABC 的外心

$\therefore$ $BO = CO$

$\angle BDO = \angle CEO$ (given)(已知)

$\angle BOD = \angle COE$ (vert. opp $\angle$s)(對頂角)

$\therefore$ $\Delta BOD \cong \Delta COE$ (AAS)

$\therefore$ $DO = EO$ ⇨ I ✓

II. $BE = BO + OE$

$= CO + DO$ (corr. sides, $\cong\Delta$s)(全等三角形對應邊)

$= CD$

$\therefore$ $BE = CD$ ⇨ II ✓

III. Not enough information to prove $\angle OBD = \angle OBC$.

沒有足夠的資料證明$\angle OBD = \angle OBC$。

22. C

$DA = AB = 10$ cm

$XA = (10\sqrt{2} - XD)$ cm

$$(10\sqrt{2} - XD)^2 = (XD)^2 + 10^2$$

$$200 - 20\sqrt{2}(XD) + (XD)^2 = (XD)^2 + 100$$

$$200 - 20\sqrt{2}(XD) = 100$$

$$20\sqrt{2}(XD) = 100$$

$$XD = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \text{ cm}$$

23. B

Let F be the mid-point of CE.

設 F 為 CE 的中點。

$OF \perp CE$ (line joining the centre and the mid-pt of chord ⊥ chord)

(圓心至弦中點的連線⊥弦)

$$\sin 30° = \frac{OF}{OD}$$

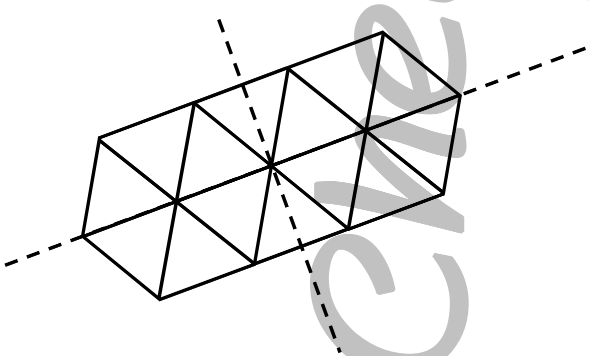
$$OF = 1$$

$OC = OB = 6$

$$CF = \sqrt{6^2 - 1^2} = \sqrt{35}$$

$$\therefore \quad CE = 2\sqrt{35}$$

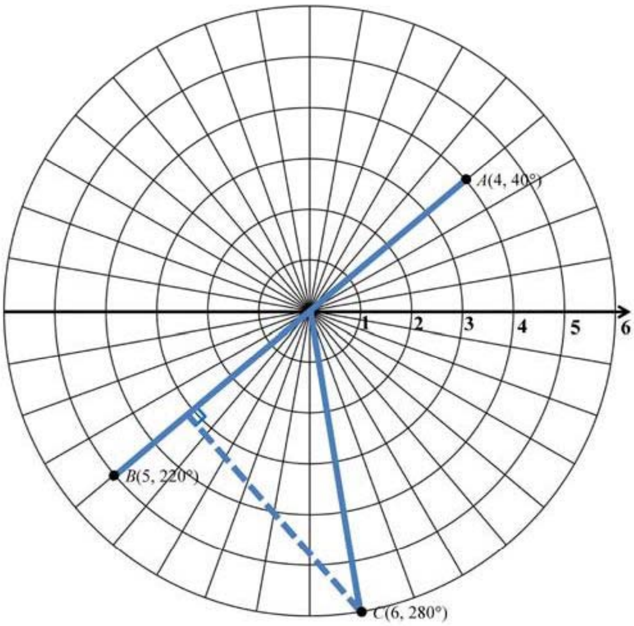
24. B



25. C

The required distance 所求距離

$= 6\sin(280° - 220°) = 6\sin 60° = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$



26. A

Let $P(x, y)$.

$$(x-6)^2 + (y+2)^2 = (y-0)^2$$
$$x^2 - 12x + 36 + y^2 + 4y + 4 = y^2$$
$$x^2 - 12x + 4y + 40 = 0$$
$$x^2 - 12x + 40 = -4y$$
$$y = -\frac{1}{4}x^2 + 3x - 10$$

27. A

Let $H(0, k)$ be the orthocentre.

設 $H(0, k)$為垂心。

$x + 2y - 12 = 0$

x-intercept = 12 ⇨ $A(12, 0)$

y-intercept = 6 ⇨ $B(0, 6)$

$$\frac{k-0}{0+2} \times \frac{6-0}{0-12} = -1$$
$$\frac{k}{2} \times -\frac{1}{2} = -1$$
$$k = 4$$

∴ Orthocentre 垂心 = (0, 4)

28. A

$25x^2 + 25y^2 + (4k + 20)x - (4k + 70)y + 175 = 0$

Centre $= (-\frac{4k+20}{2(25)}, \frac{4k+70}{2(25)}) = (-\frac{2k+10}{25}, \frac{2k+35}{25})$

Put $(-\frac{2k+10}{25}, \frac{2k+35}{25})$ into $5x + 10y + 2 = 0$,

$$5(-\frac{2k+10}{25}) + 10(\frac{2k+35}{25}) + 2 = 0$$

$$-\frac{2k+10}{5} + \frac{4k+70}{5} + 2 = 0$$

$$-2k - 10 + 4k + 70 + 10 = 0$$

$$k = -35$$

29. B

Amy 小麗	Betty 小芬
1	1, 2, 3, 4, 5, 6
2	2, 4, 6
3	3, 6
4	4
5	5
6	6

∴ The required probability 所求概率 $= \frac{14}{36} = \frac{7}{18}$

30. D

$$\frac{a+4+5+6+8+9+10+12+12+13+13+17+19+20+21+20+b}{16}=12$$

$$189+a+b=192$$

$$a+b=3$$

For $a = 0$, $b = 3$; $a = 1$, $b = 2$; $a = 2$, $b = 1$

$\therefore \quad 0 \le a \le 2$; $1 \le b \le 3$

$r = 20 + b - a$

For $a = 0$, $b = 3$: $r = 23$

For $a = 1$, $b = 2$: $r = 21$

For $a = 2$, $b = 1$: $r = 19$

$\therefore \quad 19 \le r \le 23$

31. A

$y = 2\text{f}[2(x - 1)] + 2$

$\therefore$ The graph undergoes the following transformations:

圖像進行了以下變換:

1) Translates 1 unit to the right 向右平移了 1 單位

2) Reduces $\frac{1}{2}$ time the original along x-axis 沿 x-軸縮少了原來的 $\frac{1}{2}$ 倍

3) Enlarge 2 times the original along y-axis 沿 y-軸放大了原來的 2 倍

4) Translates 2 unit upwards 向上平移了 2 單位

32. D

$2020 \times 2019 \times \log 2020 = 13480478.92$

$2019 \times 2020 \times \log 2019 = 13479601.86$

$2019 \times 2019 \times \log 2020 = 13473805.41$

$2020 \times 2020 \times \log 2019 = 13486278.24$

33. D

$$\log_{18} 18 = 1$$
$$\log_{18} (3^2 \times 2) = 1$$
$$2\log_{18} 3 + \log_{18} 2 = 1$$
$$2\log_{18} 3 + a = 1$$
$$\log_{18} 3 = \frac{1-a}{2}$$

$\log_3 2 + \log_9 4 + ... + \log_{3^n} 2^n$

$= \frac{\log_{18} 2}{\log_{18} 3} + \frac{\log_{18} 4}{\log_{18} 9} + ... + \frac{\log_{18} 2^n}{\log_{18} 3^n}$

$= \frac{\log_{18} 2}{\log_{18} 3} + \frac{2\log_{18} 2}{2\log_{18} 3} + ... + \frac{n\log_{18} 2}{n\log_{18} 3}$

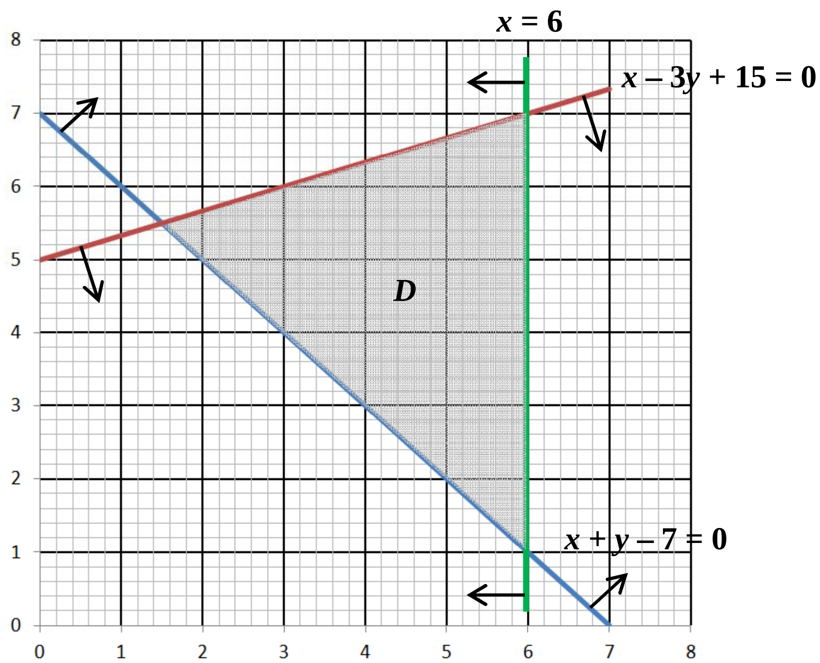
$= \frac{\log_{18} 2}{\log_{18} 3} + \frac{\log_{18} 2}{\log_{18} 3} + ... + \frac{\log_{18} 2}{\log_{18} 3}$

$= \frac{n\log_{18} 2}{\log_{18} 3}$

$= \frac{na}{\frac{1-a}{2}}$

$= \frac{2na}{1-a}$

34. C



The vertices are (6, 1), (6, 7) and (1.5, 5.5).

頂點為(6, 1)、(6, 7)及(1.5, 5.5)。

Let $f(x, y) = x - 6y$

$f(6, 1) = (6) - 6(1) = 0$

$f(6, 7) = (6) - 6(7) = -36$

$f(1.5, 5.5) = (1.5) - 6(5.5) = -31.5$

35. C

The required sum

所求總和

$= 2(7 + 8 + 9 + 10 + 11 + 12 + 13)$

$= 140$

36. B

$$y=\sqrt{1-(x-2)^2}+3$$

$$(y-3)^2=1-(x-2)^2$$

$$(x-2)^2+(y-3)^2=1$$

Centre 圓心 = (2, 3), radius 半徑 = 1

After rotation 旋轉後:

Centre 圓心 = (–3, 2), radius 半徑 = 1

∴ The minimum value 最小值

= 2 – 1

= 1

37. A

$$\frac{5k+10i}{1-2i}$$

$$=\frac{5k+10i}{1-2i}\times\frac{1+2i}{1+2i}$$

$$=\frac{5k+10ki+10i-20}{1+4}$$

$$=\frac{5k-20+(10k+10)i}{5}$$

$$=k-4+(2k+2)i$$

∴ The imaginary part 虛部 = $2k + 2$

38. C

$$2\sin^4\theta + 1 = 3\sin^2\theta$$

$$2\sin^4\theta - 3\sin^2\theta + 1 = 0$$

$$(2\sin^2\theta - 1)(\sin^2\theta - 1) = 0$$

$$\sin^2\theta = \frac{1}{2} \quad \text{or} \quad \sin^2\theta = 1$$

$$\sin\theta = \pm\frac{1}{\sqrt{2}} \quad \text{or} \quad \sin\theta = \pm 1$$

$\therefore \quad \theta = 45°, 90°, 135°, 225°, 270°, 315°$

39. D

Let $\angle BCA = a$.

$\angle PBQ = \angle QPB = \angle BCA = a$ (∠ in alt. segment)(交錯弓形的圓周角)

$\angle BPC = 90^\circ$ (∠ in semi-circle)(半圓上的圓周角)

$\angle APQ = 180^\circ - 90^\circ - a = 90^\circ - a$ (adj. ∠s on st. line)(直線上的鄰角)

$\angle ABC = 90^\circ$ (given)(已知)

$\angle QAP = 180^\circ - 90^\circ - a = 90^\circ - a = \angle APQ$ (∠ sum of Δ)(Δ內角和)

$\therefore \quad QP = QA$ (sides opp. equal ∠s)(等角對邊相等)

and $QP = QB$ (tangent properties)(切線性質)

$\therefore \quad QA = QB$

⇨ PQ bisects AB ⇨ I ✓

$\because \quad QP = QA$

$\therefore \quad \Delta APQ$ is an isosceles triangle. ⇨ II ✓

$\angle BAC = 90^\circ - a$

$\angle CBP = 180^\circ - 90^\circ - a = 90^\circ - a$ (∠ sum of Δ)(Δ內角和)

$\therefore \quad \angle BAC = \angle CBP$ ⇨ III ✓

40. B

Centre 圓心 = (0, 0)

Radius 半徑 = $\sqrt{(0+3)^2+(0-3)^2}=3\sqrt{2}$

The equation of circle 圓方程 : $x^2+y^2=18$ ……(*)

Put x = 0 into (*), $y=\pm 3\sqrt{2}$

$\therefore$ $C(0,-3\sqrt{2})$

Put y = 0 into (*), $x=\pm 3\sqrt{2}$

$\therefore$ $B(3\sqrt{2}, 0)$

$\therefore$ $AC=\sqrt{(-3-0)^2+(3+3\sqrt{2})^2}=\sqrt{36+18\sqrt{2}}$

$AB=\sqrt{(-3-3\sqrt{2})^2+(3-0)^2}=\sqrt{36+18\sqrt{2}}$

$BC=\sqrt{(0-3\sqrt{2})^2+(-3\sqrt{2}-0)^2}=6$

By Heron's formula, area of the shaded region $=9+9\sqrt{2}$

利用希羅公式，陰影部分的面積$=9+9\sqrt{2}$

41. A

$$PF=\sqrt{(\frac{\sqrt{16^2+12^2}}{2})^2+24^2}=\sqrt{10^2+24^2}=26\text{ cm}$$

$$PQ=\sqrt{(\frac{\sqrt{16^2+12^2}}{2})^2+9^2}=\sqrt{10^2+9^2}=\sqrt{181}\text{ cm}$$

$$FQ=\sqrt{(\sqrt{16^2+12^2})^2+15^2}=\sqrt{20^2+15^2}=25\text{ cm}$$

$$\cos\angle PQF=\frac{26^2+25^2-(\sqrt{181})^2}{2(26)(25)}=\frac{56}{65}$$

$$\sin\angle PQF=\sqrt{1-\cos^2\angle PQF}=\sqrt{1-(\frac{56}{65})^2}=\frac{33}{65}$$

42. A

Number of triangles can be drawn

可以繪畫三角形

$=C_3^{16}-10\times C_3^4-4C_3^3$

$=516$

43. A

The required probabiliy 所求概率 $=\dfrac{C_{26}^{48}\cdot C_3^4+C_1^4\cdot C_{25}^{48}\cdot C_3^3}{C_{26}^{52}\cdot C_3^{26}}=\dfrac{1}{5525}$

44. D

Put $y = 50$, $x = 50\sqrt{2}$

Median 中位數 $= 50\sqrt{2}$ ⇨ I ✗

Put $y = 75$, $x = 50\sqrt{3}$

Put $y = 25$, $x = 50$

Interquartile range 四分位數間距 $= 50\sqrt{3} - 50 < 50$ ⇨ II ✓

For $x > 50$, the slope of the curve is deeper, so the frequecny is higher.

當 $x > 50$，曲線的斜率較大，所以頻率較高。

Mean 平均值 > 50 ⇨ III ✓

45. B

$m_2 = 5m_1 + 2$ ⇨ I ✓

$q_2 = 5q_1$ ⇨ II ✗

$v_2 = 25v_1$ ⇨ III ✓