1. $\dfrac{\alpha^{-5}\beta^{6}}{(-\alpha\beta^{-1})^{5}}$

$=-\dfrac{\alpha^{-5}\beta^{6}}{\alpha^{5}\beta^{-5}}$ 1M

$=-\alpha^{-5-5}\beta^{6+5}$ 1M

$=-\alpha^{-10}\beta^{11}$

$=-\dfrac{\beta^{11}}{\alpha^{10}}$ 1A

2. $\dfrac{2k+h}{4k}=3h+1$

$2k+h=4k(3h+1)$ 1M

$2k+h=12hk+4k$

$h=12hk+2k$

$h=2k(6h+1)$ 1M

$k=\dfrac{h}{2(6h+1)}$ 1A

3. (a) $m^3n-2m^2n^2-3mn^3$

$=mn(m^2-2mn-3n^2)$

$=mn(m+n)(m-3n)$ 1A

(b) $4m^2-12mn-m^3n+2m^2n^2+3mn^3$

$=4m(m-3n)-mn(m+n)(m-3n)$ 1M

$=m(m-3n)[4-n(m+n)]$

$=m(m-3n)(4-mn-n^2)$ 1A

4. (a) $x-\dfrac{5x+3}{2}\geq\dfrac{8x-7}{3}$ and 及 $x+5>0$

$6x-15x-9\geq16x-14$ and 及 $x>-5$

$-25x\geq-5$ and 及 $x>-5$

$x\leq\dfrac{1}{5}$ and 及 $x>-5$ 1A + 1A

$\therefore\quad -5<x\leq\dfrac{1}{5}$ 1A

(b) 5 1A

5. Let x and y be the speeds of car A and car B respectively.

設 x 及 y 分別為汽車 A 及汽車 B 的速度。

$$\begin{cases} \frac{8}{60}x+\frac{8}{60}y=10.....(1) \\ \frac{40}{60}x-\frac{40}{60}y=10.....(2) \end{cases} \Rightarrow \begin{cases} x+y=75.....(1) \\ x-y=15.....(2) \end{cases}$$ 1A

(1) + (2):

$$2x=90$$
$$x=45$$ 1M

Put $x = 45$ into (1), $y=75-45=30$

$\therefore$ The speeds of car A and car B are 45 km/h and 30 km/h respectively. 1A + 1A

汽車 A 及汽車 B 的速度分別為 45 km/h 及 30 km/h。

6. (a) Cost 成本 $=\frac{6400}{1+60\%}=\$4000$ 1M + 1A

(b) Selling price 售價 $=\$6400(1-60\%)=\$2560<\$4000$ 1M

$\therefore$ There will be a loss.

會有虧損。

$\therefore$ Disagreed 不同意 1A

7. (a) $B(-2, 4)$, $C(4, 2)$ 1A + 1A

(b) $AB=4-(-4)=8$ units

$AC=\sqrt{(-2-4)^2+(-4-2)^2}=\sqrt{72}$ units 1M

$BC=\sqrt{(-2-4)^2+(4-2)^2}=\sqrt{40}$ units

$\therefore$ ΔABC is not an isosceles triangle. 1A

ΔABC 不是等腰三角形。

8. (a) Least possible capacity 最小可取容量 = 355 – 2.5 = 352.5 mL 1M + 1A

(b) Least possible capacity of 150 ***standard*** cans of coke

150 罐***標準***可樂的最小可取容量

= (352.5 × 150) mL 1M

= 52875 mL 1A

= 52.875 L

> 52.85 L

∴ Disagreed 不同意 1A

9. (a) $\angle ABC = \angle ADC = 20^o$

$\angle FBA = \angle AFB = 60^o$

$\angle FBC = 60^o - 20^o = 40^o$ 1M

$\angle CFB = \angle FCB = \dfrac{180° - 40°}{2} = 70°$ 1M

$\angle AFC = 70° - 60° = 10°$ 1A

(b) $\angle BCD = 180^o - 20^o = 160^o$

$\angle FCD = 360^o - 160^o - 70^o = 130^o$ 1M

$\angle CDE = 180^o - 130^o = 50^o$ 1A

10. (a) $f(x) = k_1x^2 + k_2x$ where k_1 and k_2 are non-negative integers. 1A

$f(3) = k_1(3)^2 + k_2(3) = 90 \Rightarrow 3k_1 + k_2 = 30$ ……(1)

$f(15) = k_1(15)^2 + k_2(15) = 90 \Rightarrow 15k_1 + k_2 = 6$ ……(2)

(2) – (1):

$12k_1 = -24$

$k_1 = -2$ 1M

Put $k_1 = -2$ into (1), $k_2 = 36$

$\therefore$ $f(x) = -2x^2 + 36x$ 1A

(b) $y = f(x) = -2x^2 + 36x$

Put $y = 0$,

$-2x^2 + 36x = 0$

$x^2 - 18x = 0$

$x = 0$ or $x = 18$

$\therefore$ $AD = 18$ units 1A

Put $y = -126$,

$-2x^2 + 36x = -126$ 1M

$x^2 - 18x - 63 = 0$

$x = -3$ or $x = 21$

$\therefore$ $BC = 24$ units

$\therefore$ Area 面積 $= \frac{(18+24)\times 126}{2} = 2646$ sq.units 1A

11. (a) $p(x) = (x^2 - 2x + 3)(kx + 4k - 6) + (k + 1)x - 32$ 1M

$p(-2) = 0$ 1M

$[(-2)^2 - 2(-2) + 3][k(-2) + 4k - 6] + (k + 1)(-2) - 32 = 0$

$11(2k - 6) - 2k - 2 - 32 = 0$

$22k - 66 - 2k - 34 = 0$

$20k = 100$

$k = 5$ 1A

(b) $p(x) = (x^2 - 2x + 3)(5x + 14) + 6x - 32$ 1M

$(x^2 - 2x + 3)(5x + 14) + 6x - 32 = 5x + 10$

$5x^3 + 14x^2 - 10x^2 - 28x + 15x + 42 + 6x - 32 = 5x + 10$

$5x^3 + 4x^2 - 7x + 10 = 5x + 10$

$(x + 2)(5x^2 - 6x + 5) = 5(x + 2)$

$(x + 2)(5x^2 - 6x + 5 - 5) = 0$

$x(x + 2)(5x - 6) = 0$

$x = 0$ or $x = -2$ or $x = \frac{6}{5}$ 1M

$\therefore$ The claim is incorrect. 1A

該宣稱不正確。

12. (a) $36 - (20 + r) = 11$ 1M

$r = 5$ 1A

(b) (i) $(40 + q) - (10 + p) < 33$

$q - p < 3$ 1M

$\frac{10 + p + 20 + 22 + 23 + 25 \times 2 + 28 + 29 \times 2 + 31 + 32 + 33 + 34 \times 2 + 36 \times 3 + 39 + (40 + q) \times 2}{20} = 31$ 1M

$p + 2q = 18$

$\therefore \quad \begin{cases} p = 6 \\ q = 6 \end{cases}$ or $\begin{cases} p = 8 \\ q = 5 \end{cases}$ 1A

(ii) $p = 8$ and $q = 5$ 1M

The standard deviation of the distribution

該分佈的標準差

≈ 7.341661937 1M

$\therefore$ The least possible standard deviation is 7.34. 1A

該分佈的最小可能標準差是 7.34。

13. (a) $AC : CB = 5 : 11$ 1M

$\therefore \quad C(\frac{1\times11+9\times5}{5+11}, \frac{7\times11+3\times5}{5+11}) = C(\frac{7}{2}, \frac{23}{4})$ 1A

(b) (i) $AC = \sqrt{(\frac{7}{2}-1)^2+(\frac{23}{4}-7)^2} = \frac{\sqrt{125}}{4}$

$AB = \sqrt{(9-1)^2+(3-7)^2} = \sqrt{80}$ 1M

$\because \quad r^2 = AC \times AB$

$\therefore \quad r^2 = \frac{\sqrt{125}}{4} \times \sqrt{80}$

$r^2 = 25$

$r = 5$ 1A

(ii) The equation of GC:

$y - \frac{23}{4} = \frac{1}{2}(x - \frac{7}{2})$

$4y - 23 = 2x - 7$

$x - 2y + 8 = 0$ 1M

The equation of the perpendicular bisector of AB:

AB 的垂直平分線方程

$\frac{y - \frac{7+3}{2}}{x - \frac{1+9}{2}} = -1 \div \frac{7-3}{1-9}$

$\frac{y-5}{x-5} = 2$

$y - 5 = 2x - 10$

$2x - y - 5 = 0$ 1M

$\begin{cases} x - 2y + 8 = 0 & .....(1) \\ 2x - y - 5 = 0 & .....(2) \end{cases}$

$(1)\times2-(2):$

$-3y = -21$

$y = 7$

Put $y = 7$ into (1), $x = 6$

$\therefore \quad G(6, 7)$ 1A

$\therefore$ The required equation 所求方程:

$(x-6)^2 + (y-7)^2 = 25$ 1A

14. (a) Let r be the radius of B.

設 r 為 B 的半徑。

$$\frac{4}{3}\pi r^3 = 36\pi$$

$$r^3 = 27$$ 1M

$$r = 3$$

∴ The base radius of $A = 3\times 2 = 6$ cm 1A

A 的底半徑 $= 3\times 2 = 6$ cm

(b) Let h be the height of A.

設 A 為的高度。

$$\frac{\pi(6)(\sqrt{6^2+h^2})}{4\pi(3)^2} = \frac{5}{3}$$ 1M

$$\sqrt{36+h^2} = 10$$

$$36+h^2 = 100$$

$$h = 8$$

∴ Height of $A = 8$ cm 1A

Volume of C = 3(volume of A + volume of B) 1M

$$= 3[\frac{1}{3}\pi(6)^2(8) + 36\pi]$$

$$= 3(96\pi + 36\pi)$$

$$= 396\pi \text{ cm}^3$$

Base radius of $C = 6\times\frac{3}{2} = 9$ cm

Height of $C = \frac{396\pi\times 3}{\pi(9)^2} = \frac{44}{3}$ cm

∵ $\frac{\frac{44}{3}}{8} \neq \frac{3}{2}$

∴ A and C are not similar. 1A

A 及 C 不是相似。

(c) Curved surface area of $D = 2\pi(6)(8) = 96\pi \text{ cm}^2$ 1M

Curved surface area of A + Curved surface area of B

$= \pi(6)(10) + 4\pi(3)^2$

$= 60\pi + 36\pi$

$= 96\pi \text{ cm}^2$

= Curved surface area of D

$\therefore$ Agreed 同意 1A

15. (a) Number of ways 方法數量 $= P_8^9 = 362880$ 1M+1A

(b) The required probability 所求概率 $= \dfrac{P_2^2 \times P_8^8}{P_8^9} = \dfrac{2}{9}$ 1M+1A

16. (a) $\dfrac{45-\mu}{\sigma} = -0.5 \quad \Rightarrow \quad \mu - 0.5\sigma = 45 \quad \ldots\ldots(1)$

$\dfrac{73-\mu}{\sigma} = 1.5 \quad \Rightarrow \quad \mu + 1.5\sigma = 73 \quad \ldots\ldots(2)$

(2) – (1):

$2\sigma = 28$

$\sigma = 14$

Put $\sigma = 14$ into (1),

$\mu - 0.5(14) = 45$

$\mu = 52$

$\therefore \quad \mu = 52, \ \sigma = 14$ 1A+1A

(b) Mean (μ) remains unchanged.

平均值(μ)保持不變。

Standard deviation (σ) decreased. 1M

標準差(σ)減少。

$\therefore$ Standard score of Ada will decrease. 1A

小麗的標準分減少。

Standard score of Betty will increase. 1A

美琪的標準分增加。

17. (a) Let r be the common ratio of the geometric sequence.

設 r 為該等比數列的公比。

$$\begin{cases} ar = 30 & .....(1) \\ ar^4 = 3750 & .....(2) \end{cases}$$ 1M

$\frac{(2)}{(1)}$: $r^3 = 125$

$r = 5$

Put $r = 5$ into (1),

$5a = 30$

$a = 6$

$\therefore$ $G(1) = 6$ 1A

(b) $\log_{27}(A(1)\times A(2)\times A(3)\times\ldots\times A(k)) > 20212022$

$\frac{\log_3(A(1)\times A(2)\times A(3)\times\ldots\times A(k))}{\log_3 27} > 20212022$ 1M

$\log_3(A(1)\times A(2)\times A(3)\times\ldots\times A(k)) > 60636066$

$\log_3 A(1) + \log_3 A(2) + \log_3 A(3) + \ldots + \log_3 A(k) > 60636066$ 1M

$G(1) + G(2) + G(3) + \ldots + G(k) > 60636066$

$\frac{6(5^k - 1)}{5 - 1} > 60636066$ 1M

$5^k > 40424045$

$k > \frac{\log 40424045}{\log 5}$

$k > 10.8826412$

$\therefore$ The least value of k is 11. 1A

k 的最小值為 11。

18. (a) $f(x) = x^2 - 16kx + 4x + 2k^2 + 36k + 32$

$f(x) = x^2 - (16k - 4)x + 2k^2 + 36k + 32$

$f(x) = [x - (8k - 2)]^2 - (8k - 2)^2 + 2k^2 + 36k + 32$ 1M

$f(x) = [x - (8k - 2)]^2 - 64k^2 + 32k - 4 + 2k^2 + 36k + 32$

$f(x) = [x - (8k - 2)]^2 - 62k^2 + 68k + 28$ 1A

$\therefore$ $P = (8k - 2, -62k^2 + 68k + 28)$ 1A

(b) $Q = (40k - 30, -186k^2 + 204k + 175)$ 1A

(c) $\because$ $PQRS$ is a rhombus

$PQRS$ 為菱形。

$\therefore$ $PQ = PS$ 1M

$(8k - 2 - 40k + 30)^2 + (-62k^2 + 68k + 28 + 186k^2 - 204k - 175)^2$

$= (8k - 2 - 54)^2 + (-62k^2 + 68k + 28 + 186k^2 - 204k - 327)^2$

$(-32k + 28)^2 + (124k^2 - 136k - 147)^2 = (8k - 56)^2 + (124k^2 - 136k - 299)^2$

$(-32k + 28)^2 + (124k^2 - 136k - 299 + 152)^2 = (8k - 56)^2 + (124k^2 - 136k - 299)^2$

$(-32k + 28)^2 + (124k^2 - 136k - 299)^2 + 2(124k^2 - 136k - 299)(152) + (152)^2$

$= (8k - 56)^2 + (124k^2 - 136k - 299)^2$ 1M

$(-32k + 28)^2 + 2(124k^2 - 136k - 299)(152) + (152)^2 = (8k - 56)^2$

$(-8k + 7)^2 + 19(124k^2 - 136k - 299) + 1444 = (2k - 14)^2$

$64k^2 - 112k + 49 + 2356k^2 - 2584k - 5681 + 1444 = 4k^2 - 56k + 196$

$2416k^2 - 2640k - 4384 = 0$

$k = 2$ or $k = -\frac{137}{151}$ (rej.) 1A

$\therefore$ When $k = 2$, $PQ = PS$

$\therefore$ There is a point R such that $PQRS$ is a rhombus. 1A

有一點 R 使得 PQRS 為一菱形。

(英文版題目適用)

19. (a) (i) $\angle BCA = 180^o - 110^o - 40^o = 30^o$

Consider ΔBCD,

$$\frac{BD}{\sin 40°} = \frac{9}{\sin 55°}$$

$$BD = \frac{9}{\sin 55°} \times \sin 40° = 7.0622890199 \text{ cm} \quad \text{1A}$$

Consider ΔABD,

$$\frac{AD}{\sin 55°} = \frac{BD}{\sin 30°}$$

$$AD = \frac{7.0622890199}{\sin 30°} \times \sin 55° = 11.570176981 \text{ cm} \approx 11.6 \text{ cm (cor. to 3 sig. fig.)} \quad \text{1A}$$

(ii) Area

$$= \frac{1}{2}(CD)(BD)\sin \angle BDC$$

$$= \frac{1}{2}(9)(7.062289019)\sin(180° - 55° - 40°) \quad \text{1M}$$

$$= 31.65936695 \text{ cm}^2 \approx 31.7 \text{ cm}^2 \text{ (cor. to 3 sig. fig.)} \quad \text{1A}$$

(b) (i) Consider ΔABD,

$$(AB)^2 = (AD)^2 + (BD)^2 - 2(AD)(BD)\cos \angle ADB$$

$$(AB)^2 = (11.570176981)^2 + (7.0622890199)^2 - 2(11.570176981)(7.0622890199)\cos 95°$$

$$AB = 14.07082975 \text{ cm}$$

Consider ΔBCD,

$$(BC)^2 = (BD)^2 + (CD)^2 - 2(BD)(CD)\cos \angle BDC$$

$$(BC)^2 = (7.0622890199)^2 + 9^2 - 2(7.0622890199)(9)\cos 85°$$

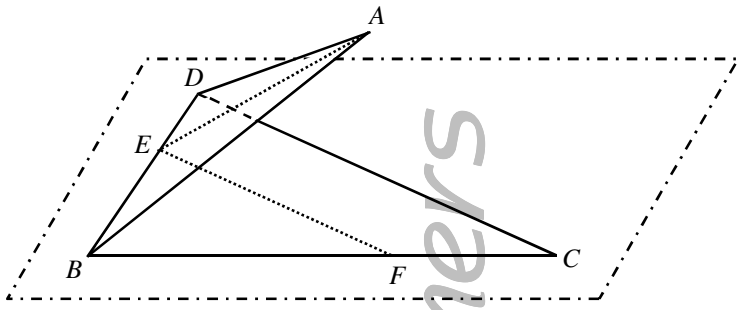
$$BC = 10.94516256 \text{ cm}$$

Consider ΔABC

$$\cos \angle ABC = \frac{(AB)^2 + (BC)^2 - (AC)^2}{2(AB)(BC)} = \frac{(14.07082975)^2 + (10.94516256)^2 - (6)^2}{2(14.07082975)(10.94516256)} \quad \text{1M}$$

$$\angle ABC = 23.81682084° \approx 23.8° \text{ (cor. to 3 sig. fig.)} \quad \text{1A}$$

(b) (ii)



In the figure, draw the line AE and EF such that $AE \perp BD$ and $EF \perp BD$.

Consider ΔABE,

$AE = AB\sin 55° = 8.513825089\sin 55° = 6.974117226$ cm

$\because$ $\Delta ABE \cong \Delta FBE$

$\therefore$ $EF = AE = 6.974117226\,\text{cm}$ & $BF = AB = 8.513825089\,\text{cm}$ 1M

Consider ΔABF,

$(AF)^2 = (AB)^2 + (BF)^2 - 2(AB)(BF)\cos\angle ABF$

$(AF)^2 = (8.513825089)^2 + (8.513825089)^2 - 2(8.513825089)(8.513825089)\cos 33.01217209°$

$AF = 4.837848128\,\text{cm}$ 1M

$\therefore$ The required angle $= \angle AEF$

$$= \cos^{-1}\frac{(AE)^2 + (EF)^2 - (AF)^2}{2(AE)(EF)}$$ 1M

$= 40.58868567°$

$\approx 40.6°$ (cor. to 3 sig. fig.) 1A

(iii) Height of the tetrahedron $ABCD$

$= AE\sin\angle AEF$

$= 6.974117226\sin 40.58868567°$

$= 4.537529925\,\text{cm}$ 1A

Volume of the tetrahedron $ABCD$

$= \frac{1}{3}\times(24.62661575)\times(4.537529925)$

$\approx 37.2\,\text{cm}^3$ (cor. to 3 sig. fig.) 1A

(中文版題目適用)

19. (a) (i) $\angle BCA = 180^o - 110^o - 40^o = 30^o$

考慮 ΔBCD,

$$\frac{BD}{\sin 30°} = \frac{9}{\sin 55°}$$

$$BD = \frac{9}{\sin 55°} \times \sin 30° = 5.493485649 \text{ cm} \quad \text{1A}$$

考慮ΔABD,

$$\frac{AD}{\sin 55°} = \frac{BD}{\sin 40°}$$

$$AD = \frac{5.493485649}{\sin 40°} \times \sin 55° = 7.000757221 \text{ cm} \approx 7.00 \text{ cm (cor. to 3 sig. fig.)} \quad \text{1A}$$

(ii) 面積

$$= \frac{1}{2}(CD)(BD)\sin \angle BDC$$

$$= \frac{1}{2}(9)(5.493485649)\sin(180° - 55° - 30°) \quad \text{1M}$$

$$= 24.62661575 \text{ cm}^2 \approx 24.6 \text{ cm}^2 \text{ (cor. to 3 sig. fig.)} \quad \text{1A}$$

(b) (i) 考慮 ΔABD,

$$(AB)^2 = (AD)^2 + (BD)^2 - 2(AD)(BD)\cos \angle ADB$$

$$(AB)^2 = (7.000757221)^2 + (5.493485649)^2 - 2(7.000757221)(5.493485649)\cos 85°$$

$$AB = 8.513825089 \text{ cm}$$

考慮 ΔBCD,

$$(BC)^2 = (BD)^2 + (CD)^2 - 2(BD)(CD)\cos \angle BDC$$

$$(BC)^2 = (5.493485649)^2 + 9^2 - 2(5.493485649)(9)\cos 95°$$

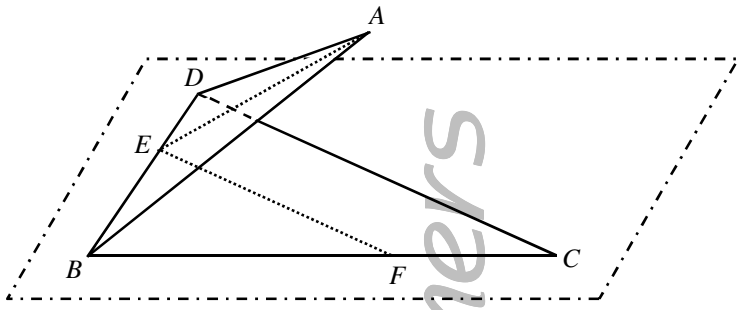
$$BC = 10.94516256 \text{ cm}$$

考慮 ΔABC

$$\cos \angle ABC = \frac{(AB)^2 + (BC)^2 - (AC)^2}{2(AB)(BC)} = \frac{(8.513825089)^2 + (10.94516256)^2 - (6)^2}{2(8.513825089)(10.94516256)} \quad \text{1M}$$

$$\angle ABC = 33.01217209° \approx 33.0° \text{ (cor. to 3 sig. fig.)} \quad \text{1A}$$

(b) (ii)



圖中，繪畫直線 AE 及 EF 使得 $AE \perp BD$ 及 $EF \perp BD$。

考慮 ΔABE,

$AE = AB\sin 55° = 8.513825089\sin 55° = 6.974117226$ cm

$\because$ $\Delta ABE \cong \Delta FBE$

$\therefore$ $EF = AE = 6.974117226\,\text{cm}$ & $BF = AB = 8.513825089\,\text{cm}$ 1M

考慮 ΔABF,

$(AF)^2 = (AB)^2 + (BF)^2 - 2(AB)(BF)\cos\angle ABF$

$(AF)^2 = (8.513825089)^2 + (8.513825089)^2 - 2(8.513825089)(8.513825089)\cos 33.01217209°$

$AF = 4.837848128\,\text{cm}$ 1M

$\therefore$ 所求角度 $= \angle AEF$

$$= \cos^{-1}\frac{(AE)^2 + (EF)^2 - (AF)^2}{2(AE)(EF)}$$ 1M

$= 40.58868567°$

$\approx 40.6°$ (cor. to 3 sig. fig.) 1A

(iii) 四面體 $ABCD$ 的高

$= AE\sin\angle AEF$

$= 6.974117226\sin 40.58868567°$

$= 4.537529925\,\text{cm}$ 1A

四面體 $ABCD$ 的體積

$$= \frac{1}{3}\times(24.62661575)\times(4.537529925)$$

$\approx 37.2\,\text{cm}^3$ (cor. to 3 sig. fig.) 1A