F.6 Mathematics 2022 Mock Exam Paper II

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Paper 2 Answer:

1.	B	26.	D
2.	D	27.	D
3.	D	28.	C
4.	C	29.	A
5.	D	30.	C
6.	B	31.	D
7.	A	32.	D
8.	B	33.	A
9.	A	34.	B
10.	C	35.	B
11.	C	36.	A
12.	B	37.	C
13.	C	38.	B
14.	A	39.	D
15.	C	40.	A
16.	A	41.	D
17.	D	42.	B
18.	C	43.	B
19.	C	44.	B
20.	D	45.	C
21.	A		
22.	B		
23.	D		
24.	A		
25.	C		

1. B

$$\frac{(4^n)(64^{3n})}{4096^n} = \frac{4^n \times 4^{9n}}{4^{6n}} = 4^{n+9n-6n} = 4^{4n} = 16^{2n}$$

2. D

$$a(a-b) = b(1+a)$$
$$a^2 - ab = b + ab$$
$$a^2 = b + 2ab$$
$$a^2 = b(1+2a)$$
$$b = \frac{a^2}{2a+1}$$

3. D

$$(a-2)(a+b)(a-b)$$
$$= (a-2)(a^2-b^2)$$
$$= a^3 - ab^2 - 2a^2 + 2b^2$$
$$= a^3 - 2a^2 - ab^2 + 2b^2$$

4. C

$$\frac{8}{k-8} - \frac{7}{7-k}$$
$$= \frac{8}{k-8} + \frac{7}{k-7}$$
$$= \frac{8(k-7)+7(k-8)}{(k-7)(k-8)}$$
$$= \frac{8k-56+7k-56}{(k-7)(k-8)}$$
$$= \frac{15k-112}{(k-7)(k-8)}$$

5. D

$$20.22 - 0.005 \le x < 20.22 + 0.005$$
$$20.215 \le x < 20.225$$

6. B

Put $x = -5$

$$p[3(-5)-4]+q[(-5)+6]=r[(-5)+5]$$
$$p(-19)+q=0$$
$$19p=q$$
$$\frac{p}{q}=\frac{1}{19}$$

$\therefore$ $p : q = 1 : 19$

7. A

$f(x) = (x - 5)(x + a) + b$

$f(-3) = (-3 - 5)(-3 + a) + b = 24 - 8a + b = 5$ ……(1)

$f(7) = (7 - 5)(7 + a) + b = 14 + 2a + b = 5$ ……(2)

By solving (1) and (2), $a = 1$, $b = -11$

8. B

$f(x) = (x^2 - 9)Q(x) + ax + b$ where $Q(x)$ is the quotient, a and b are constants.

$f(x) = (x + 3)(x - 3)Q(x) + ax + b$

$f(-3) = (-3 + 3)(-3 - 3)Q(-3) + a(-3) + b$

$-3a + b = -6$ ……(1)

$f(3) = (3 + 3)(3 - 3)Q(3) + a(3) + b$

$3a + b = 0$ ……(2)

By solving (1) and (2), $a = 1$, $b = -3$

$\therefore$ Remainder $= x - 3$

9. A

Let n be the number of blue balls.

$$\frac{\frac{7n}{3}-20}{n+\frac{7n}{3}-20}=\frac{1}{2}$$

$$\frac{14n}{3}-40=\frac{10n}{3}-20$$

$$\frac{4n}{3}=20$$

$$n=15$$

$\therefore$ The number of blue balls is 15.

10. C

$7x+5<4(x-1)$

$7x+5<4x-4$

$3x<-9$

$x<-3$

$1-2x>5$

$-2x>4$

$x<-2$

$\because$ $x<-3$ or $x<-2$

$\therefore$ $x<-2$

11. C

$$\frac{5a+4b}{4a+5b}=\frac{8}{9}$$

$$45a+36b=32a+40b$$

$$13a=4b$$

$$\frac{a}{b}=\frac{4}{13}$$

$$\frac{5a+6b}{6a+5b}=\frac{5(4)+6(13)}{6(4)+5(13)}=\frac{98}{89}$$

12. B

$$y = \frac{k\sqrt{x}}{\sqrt[3]{z}}$$

$$y^6 = \frac{kx^3}{z^2}$$

$$k = \frac{y^6 z^2}{x^3}$$

$$\frac{1}{k} = \frac{x^3}{y^6 z^2} = \text{constant}$$

13. C

1st pattern : $1^2 + 4$

2rd pattern : $3^2 + 4$

3th pattern : $5^2 + 4$

$\therefore$ 7th pattern : $13^2 + 4 = 173$

14. A

$y = -3 - 2(1 - x)^2$

Vertex = (1, –3)

$\therefore$ I ✓

y-intercept $= -3 - 2(1 - 0)^2 = -5$

$\therefore$ II ✗

Line of symmetry is $x = 1$

$\therefore$ III ✗

15. C

$$\text{Height of the prism} = \frac{1152}{8 \times \frac{1}{2} \times 16 \times \frac{8}{\tan 22.5°}} = 0.931980515\text{ cm}$$

$$\text{Total surface area of the prism} = 16 \times 0.931980515 \times 8 + 2 \times 8 \times \frac{1}{2} \times 16 \times \frac{8}{\tan 22.5°} = 2591\text{ cm}^2$$

16. A

Let r and h be the base radius of the sphere and the height of the cone respectively.

The base radius of the cone = $3r$

$$\frac{r}{3r}=\frac{h-r}{\sqrt{h^2+(3r)^2}}$$

$$\frac{1}{3}=\frac{h-r}{\sqrt{h^2+9r^2}}$$

$$\frac{1}{9}=\frac{(h-r)^2}{h^2+9r^2}$$

$$h^2+9r^2=9h^2-18hr+9r^2$$

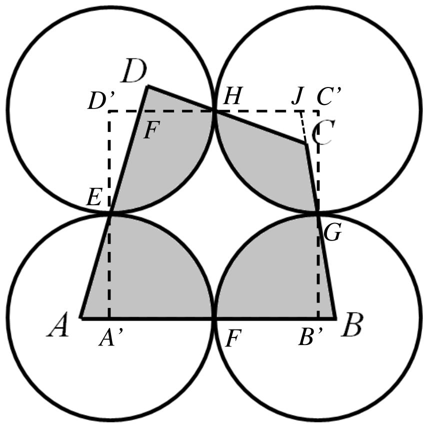
$$8h^2-18hr=0$$

$$2h(4h-9r)=0$$

$$h=0\text{ (rej.)}\qquad\text{or}\qquad \frac{h}{r}=\frac{9}{4}$$

$\therefore$ The required ratio = $h : 3r = 9 : 12 = 3 : 4$

17. D



Let A', B', C' and D' be the centres of the four circles respectively.

$\Delta AA'E\cong\Delta FD'E$, $\Delta BB'G\cong\Delta JC'G$

$JC'=BB'=\tan 10°$, $HJ=1-\tan 10°$

$$\frac{JC}{\sin 20°}=\frac{HJ}{\sin 60°}$$

$$JC=\frac{(1-\tan 10°)\sin 20°}{\sin 60°}$$

Area of $\Delta JHC=\frac{1}{2}(HJ)(JC)\sin\angle HJC=\frac{1}{2}(1-\tan 10°)[\frac{(1-\tan 10°)\sin 20°}{\sin 60°}]\sin 100°=0.131932622$

$FH=1-FD'=1-\tan 20°$, $DH=\cos 20°(1-\tan 20°)$

Area of $\Delta DFH=\frac{1}{2}(FH)(DH)\sin\angle DHF=\frac{1}{2}(1-\tan 20°)[\cos 20°(1-\tan 20°)]\sin 20°=0.065007338$

Area of the shaded region

$=\pi(1)^2-0.131932622+0.065007338$

$=3.07466737$

$\approx 3.075\text{ cm}^2$

18. C

$BD = \sqrt{157^2 - 132^2} = 85\text{ cm}$

$AD = \sqrt{85^2 - 84^2} = 13\text{ cm}$

$\angle EDA = 180° - \sin^{-1}\dfrac{84}{85} - \sin^{-1}\dfrac{132}{157} = 41.57649138°$

$\sin\angle EDA = \dfrac{AE}{AD}$

$AE = 13\sin 41.57649138° = 8.62705133 \approx 9\text{ cm}$

19. C

$\because$ $AE = EB$, $DF : FC = 1:3$

$\therefore$ $AE = 2$, $EB = 2$, $DF = 1$, $FC = 3$

$\because$ $\Delta AEG \sim \Delta FDG$ (A.A.A) & $\Delta EBH \sim \Delta CFH$ (A.A.A)

$\therefore$ Height of $\Delta EBH = \dfrac{2}{2+3}\times 4 = \dfrac{8}{5}$

$\therefore$ Height of $\Delta AEG = \dfrac{2}{3}\times 4 = \dfrac{8}{3}$

$\therefore$ $\text{Area} = 4\times 4\times\dfrac{1}{2} - \dfrac{1}{2}\times 2\times\dfrac{8}{5} - \dfrac{1}{2}\times 2\times\dfrac{8}{3} = \dfrac{56}{15}\text{ cm}^2$

20. D

$\angle BCD = \angle CDE = \dfrac{180° - 48°}{2} = 66°$

$\angle BEC = \dfrac{180° - 48° - 66°}{2} = 33°$

$\angle DFE = 33° + 48° = 81°$

21. A

$\angle ADE = \angle ECF$ (given)

$= \angle ACB$ (vert. opp. $\angle$)

$= \angle BAC$ (base $\angle$s, isos. Δ)

$\therefore$ $ED = EA$

$\therefore$ ΔADE is an isosceles triangle.

$\therefore$ I ✓

Let $\angle BDF = \angle ECF = \angle ACB = \angle BAC = x$

$\angle AED = x + 15°$

$x + 15° + x + x = 180°$

$3x = 165°$

$x = 55°$

$\therefore$ $\angle BDF = 55°$

$\therefore$ II ✓

$\because$ $\Delta CEF \sim \Delta DBF$

$\therefore$ $\dfrac{CE}{DB} = \dfrac{CF}{DF}$

$\therefore$ III ×

22. B

$\angle CBA = 90°$ ($\angle$ in semi-circle)

$\angle CBE = 90° - 25° = 65°$

$\angle CBE = \angle CAE = 65°$ ($\angle$s in the same segment)

$\angle DAE = 65° - 37° = 28°$

23. D

$P(-2, 5)$ $\Rightarrow$ $Q(5, 2)$ $\Rightarrow$ $R(5, -3)$

24. A

$$CD = AC\sin\alpha$$

$$AB = \frac{AC}{\sin\beta}$$

$$\frac{CD}{AB} = \frac{AC\sin\alpha}{\frac{AC}{\sin\beta}} = \sin\alpha\sin\beta$$

25. C

$$\begin{aligned} PQ &= EF \\ (x-3)^2 + (y-0)^2 &= (1+2)^2 + (-4-1)^2 \\ (x-3)^2 + y^2 &= 34 \\ x^2 - 6x + 9 + y^2 &= 34 \\ x^2 + y^2 - 6x - 25 &= 0 \end{aligned}$$

26. D

$$P(\frac{3\times2+8\times3}{3+2}, \frac{5\times2+10\times3}{3+2}) = P(6,8)$$

The required equation is

$$\begin{aligned} \frac{y-4}{x-11} &= \frac{8-4}{6-11} \\ \frac{y-4}{x-11} &= -\frac{4}{5} \\ 5y-20 &= -4x+44 \\ 4x+5y-64 &= 0 \end{aligned}$$

27. D

$$\begin{cases} x^2+y^2+2x+6y-30=0 & \text{......(1)} \\ x-2y-3=0 & \text{......(2)} \end{cases}$$

From (2): $x=2y+3$ ……(3)

Sub. (3) into (1),

$$\begin{aligned}(2y+3)^2+y^2+2(2y+3)+6y-30&=0\\ 4y^2+12y+9+y^2+4y+6+6y-30&=0\\ 5y^2+22y-15&=0\\ y=\frac{3}{5} \quad \text{or} \quad y&=-5\end{aligned}$$

$\therefore$ $P(\frac{21}{5},\frac{3}{5})$, $Q(-7,-5)$

Mid-point of $PQ=(-\frac{7}{5},-\frac{11}{5})$

The required equation is

$$\begin{aligned}(x+\frac{7}{5})^2+(y+\frac{11}{5})^2&=(\frac{1}{2}\sqrt{(\frac{21}{5}+7)^2+(\frac{3}{5}+5)^2})^2\\ x^2+\frac{14}{5}x+\frac{49}{25}+y^2+\frac{22}{5}y+\frac{121}{25}&=\frac{196}{5}\\ x^2+y^2+\frac{14}{5}x+\frac{22}{5}y-\frac{162}{5}&=0\\ 5x^2+5y^2+14x+22y-162&=0\end{aligned}$$

28. C

The required probability

$=\frac{16}{20}$

$=\frac{4}{5}$

29. A

$$m_1 = \frac{a+(a+1)+(a+2)}{3} = \frac{3a+3}{3} = a+1$$

$$m_2 = \frac{(b-1)+(b+1)+(b+3)}{3} = \frac{3b+3}{3} = b+1$$

$\because \quad a < b$

$\therefore \quad a+1 < b+1$

$\therefore \quad m_1 < m_2$

$$\sigma_1 = \sqrt{\frac{[a-(a+1)]^2+[(a+1)-(a+1)]^2+[(a+2)-(a+1)]^2}{3}} = \sqrt{\frac{2}{3}}$$

$$\sigma_2 = \sqrt{\frac{[(b-1)-(b+1)]^2+[(b+1)-(b+1)]^2+[(b+3)-(b+1)]^2}{3}} = \sqrt{\frac{8}{3}}$$

$\therefore \quad \sigma_1 < \sigma_2$

30. C

19, $x-3$, $x+2$, $x+7$, $x+19$

$x+2 = 15$

$x = 13$ (rej.)

$x-3$, $x+2$, $x+7$, 19, $x+19$

$x+7 = 15$

$x = 8$

$$\text{mean} = \frac{5+10+15+19+27}{5} = 15.2$$

31. D

$$\begin{aligned}
5\times 2^8+17-48\times 2^3 &= (2^2+1)\times 2^8+(2^4+1)-(2^5+2^4)\times 2^3 \\
&= 2^{10}-2^7+2^4+1 \\
&= 2^7(2^3-1)+2^4+1 \\
&= 2^7(7)+2^4+1 \\
&= 2^7(2^2+2+1)+2^4+1 \\
&= 2^9+2^8+2^7+2^4+1 \\
&= 1110010001_2
\end{aligned}$$

32. D

$h^2+4h=k^2+4k=3$

$\therefore \quad h+k=-4\,,\;\; hk=-3$

$(\frac{2}{2^h})^{1-k}=(2^{1-h})^{1-k}=2^{(1-h)(1-k)}=2^{1-(h+k)+hk}=2^{1+4-3}=2^2=4$

33. A

log 2 : log 4 : log x = log y : log 16 : log 9

$\therefore \quad \frac{\log 2}{\log y}=\frac{\log 4}{\log 16}=\frac{\log x}{\log 9}$

$\frac{\log 2}{\log y}=\frac{1}{2}=\frac{\log x}{\log 9}$

$\therefore \quad \log x=\frac{\log 9}{2}\Rightarrow x=3$

$\log y=2\log 2\Rightarrow y=4$

$\therefore \quad xy=(3)(4)=12$

34. B

$$\frac{3}{2^x+1}+\frac{2}{2^x-1}=3$$

$$\frac{3(2^x-1)+2(2^x+1)}{(2^x+1)(2^x-1)}=3$$

$$5(2^x)-1=3(2^x+1)(2^x-1)$$

$$5(2^x)-1=3(2^x)^2-3$$

$$3(2^x)^2-5(2^x)-2=0$$

$$2^x=2 \quad \text{or} \quad 2^x=-\frac{1}{3}\text{ (rej.)}$$

$$x=1$$

$\therefore$ I ✗

II ✗

III ✓

35. B

36. A

Sum of roots $= 4-3i+4+3i = 8$

Product of roots $= (4-3i)(4+3i) = 25$

$\therefore \quad b = -8\,,\ c = 25$

$\therefore \quad b-c = -8-25 = -33$

37. C

Let r be the common ratio of the geometric sequence.

$$a_1 r^4 = 126 \text{......(1)}$$
$$a_1 r^6 = 56 \text{ ......(2)}$$

$\dfrac{(2)}{(1)}: \quad r^2 = \dfrac{4}{9} \Rightarrow r = \pm\dfrac{2}{3} \Rightarrow a_1 = \dfrac{5103}{8}$

For $r = -\dfrac{2}{3}$, $a_8 = a_1 r^7 = \dfrac{5103}{8}(-\dfrac{2}{3})^7 = -37\dfrac{1}{3}$

$\therefore$ I ✗

$\dfrac{a_4}{a_6} = \dfrac{1}{r^2} = \dfrac{9}{4}$

$\therefore$ II ✓

For $r = \dfrac{2}{3}$, $a_1 + a_2 + a_3 + \dots + a_{2n} = \dfrac{\frac{5103}{8}[1-(\frac{2}{3})^{2n}]}{1-\frac{2}{3}} = \dfrac{15309}{8}[1-(\dfrac{2}{3})^{2n}]$

$$a_2 + a_4 + a_6 + \dots + a_{2n} = \dfrac{\frac{1701}{4}[1-(\frac{4}{9})^{n}]}{1-\frac{4}{9}} = \dfrac{15309}{20}[1-(\dfrac{2}{3})^{2n}]$$

$\therefore \quad a_1 + a_2 + a_3 + \dots + a_{2n} > a_2 + a_4 + a_6 + \dots + a_{2n}$

For $r = -\dfrac{2}{3}$, $a_1 + a_2 + a_3 + \dots + a_{2n} = \dfrac{\frac{5103}{8}[1-(-\frac{2}{3})^{2n}]}{1+\frac{2}{3}} = \dfrac{15309}{40}[1-(\dfrac{2}{3})^{2n}]$

$$a_2 + a_4 + a_6 + \dots + a_{2n} = \dfrac{-\frac{1701}{4}[1-(\frac{4}{9})^{n}]}{1-\frac{4}{9}} = -\dfrac{15309}{20}[1-(\dfrac{2}{3})^{2n}]$$

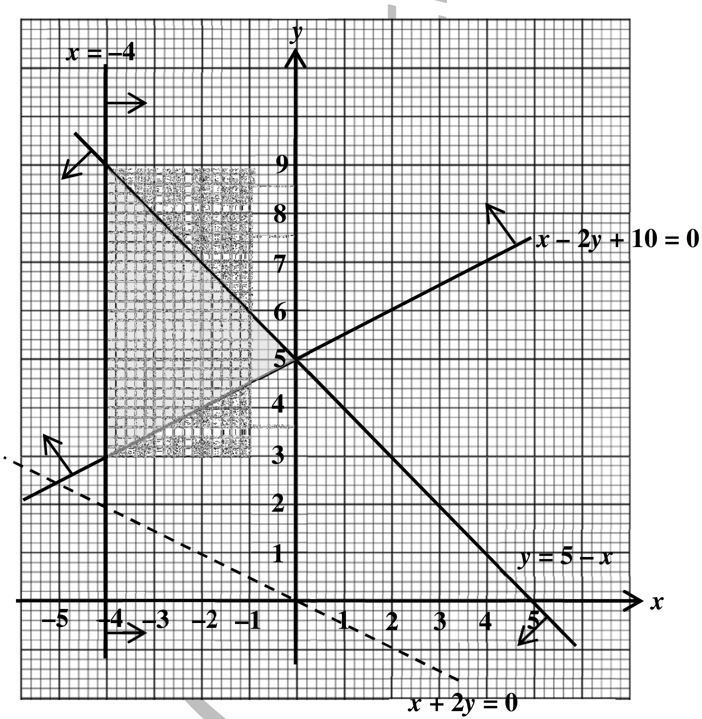
$\therefore \quad a_1 + a_2 + a_3 + \dots + a_{2n} > a_2 + a_4 + a_6 + \dots + a_{2n}$

$\therefore$ III ✓

38. B

$$x^2 - bx + cx - bc \le 0$$
$$x(x-b) + c(x-b) \le 0$$
$$(x-b)(x+c) \le 0$$
$$-c \le x \le b$$

39. D



From the graph, the maximum point is (–4, 9).

$\therefore$ Maximum value of $x + 2y = (-4) + 2(9) = 14$

$\therefore$ $k = 14$

40. A

Let r be the radius of the circle and $s=\dfrac{12+18+25}{2}=27.5$

Consider the area of ΔABC,

$$\begin{aligned}\sqrt{27.5(27.5-12)(27.5-18)(27.5-25)}&=\frac{1}{2}(12+18)r\\ r^2&=\frac{6479}{144}\\ r&=\frac{\sqrt{6479}}{12}\end{aligned}$$

$$\cos\angle BAC=\frac{12^2+25^2-18^2}{2(12)(25)}=\frac{89}{120}$$

$$\therefore\quad \sin\angle BAC=\frac{\sqrt{120^2-89^2}}{120}=\frac{\sqrt{6497}}{120}$$

$$\therefore\quad AO=\frac{r}{\sin\angle BAC}=\frac{\frac{\sqrt{6479}}{12}}{\frac{\sqrt{6497}}{120}}=10\text{ cm}$$

41. D

Put (0, 1),

A. $a=4$ and $k=2$ $\Rightarrow$ $1=4\sin^2 2(0)+b$ $\Rightarrow$ $b=1$

B. $a=4$ and $k=\dfrac{1}{2}$ $\Rightarrow$ $1=4\sin^2\dfrac{1}{2}(0)+b$ $\Rightarrow$ $b=1$

C. $a=3$ and $k=2$ $\Rightarrow$ $1=3\sin^2 2(0)+b$ $\Rightarrow$ $b=1$

D. $a=3$ and $k=\dfrac{1}{2}$ $\Rightarrow$ $1=3\sin^2\dfrac{1}{2}(0)+b$ $\Rightarrow$ $b=1$

Put (180, 4),

A. $a=4$ and $k=2$ $\Rightarrow$ $4=4\sin^2 2(180)+b$ $\Rightarrow$ $b=4$

B. $a=4$ and $k=\dfrac{1}{2}$ $\Rightarrow$ $4=4\sin^2\dfrac{1}{2}(180)+b$ $\Rightarrow$ $b=0$

C. $a=3$ and $k=2$ $\Rightarrow$ $4=3\sin^2 2(180)+b$ $\Rightarrow$ $b=4$

D. $a=3$ and $k=\dfrac{1}{2}$ $\Rightarrow$ $4=3\sin^2\dfrac{1}{2}(180)+b$ $\Rightarrow$ $b=1$

$\therefore$ D is the answer.

42. B

$VR = \sqrt{10^2 - 3^2} = \sqrt{91}$

$CR = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$

$\cos\angle VCR = \dfrac{10^2 + (3\sqrt{5})^2 - (\sqrt{91})}{2(10)(3\sqrt{5})} = \dfrac{9}{10\sqrt{5}}$

$PR^2 = (3\sqrt{5})^2 + 5^2 - 2(3\sqrt{5})(5)\cos\angle VCR = (3\sqrt{5})^2 + 5^2 - 2(3\sqrt{5})(5) \times \dfrac{9}{10\sqrt{5}}$

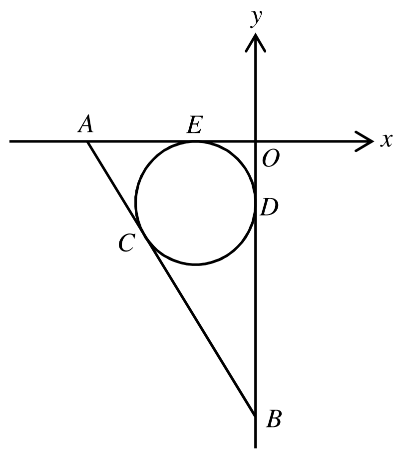
$PR = \sqrt{43}$

$PQ = PR = \sqrt{43}$

$QR = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$\cos\angle QPR = \dfrac{(\sqrt{43})^2 + (\sqrt{43})^2 - (3\sqrt{2})^2}{2(\sqrt{43})(\sqrt{43})} = \dfrac{34}{43}$

43. B



Let $B(0, -b)$

$OE = OD = 1$

$BD = BC = b - 1$

$AE = AC = 3 - 1 = 2$

$AC + BC = AB$

$2 + b - 1 = \sqrt{3^2 + b^2}$

$b^2 + 2b + 1 = 9 + b^2$

$b = 4$

$\therefore$ $B(0, -4)$

44. B

The required probability $= \dfrac{2! \times 2! \times 2}{4!} = \dfrac{1}{3}$

45. C

Standard deviation $= \sqrt{4084441\sigma^2} = 2021\sigma$

$\therefore$ Each datum of $\{a_1, a_2, a_3, \ldots, a_{2021}\}$ is multiplied by 2021 to get a new standard deviation of 2021σ.

$\therefore$ $\{2021a_1, 2021a_2, 2021a_3, \ldots, 2021a_{2021}\}$