

Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

評卷參考

本文件供閱卷員參考而設，並不應被視為標準答案。考生以及沒有參與評卷工作的教師在詮釋文件內容時應小心謹慎。

一般閱卷原則

1. 評卷時，閱卷員須跟評卷參考的評分標準給分，這是十分重要的。很多時考生會運用評卷參考以外的方法而得到正確答案，一般來說，只要運用合理的方法取得正確答案，該考生應可獲得該部分的**所有分數**(除題目特別指明特定方法外)。閱卷員應有耐性地評閱評卷參考以外的解題方法。
2. 在評卷參考中，分數會分為下列三類：

「M」分	使用正確方法的得分；
「A」分	正確答案的得分；
沒有「M」或「A」的分	正確地完成證題或推演得題目所給的答案的得分。

某些題目由數部分組成，而較後部分的答案卻需依賴較前部分所得的結果。在這情況下，若考生因為前部分錯誤的結果而導致後部分的答案錯誤，但卻能運用正確的方法去解題，則方法正確的步驟可給「M」分，而相應的答案將沒有「A」分(除特別指明外)。
3. 為方便閱卷員評卷，評卷參考已盡量詳盡。當然，考生的答案多不會如評卷參考般清楚列寫出來，諸如欠缺某幾個步驟或將步驟隱含於字裏行間。如遇到類似情況，閱卷員應運用他們的專業知識去判斷是否給分。一般來說，如考生的答案顯示他已運用相關的概念或技巧，則該部分應予給分。
4. 評卷時遇有不清楚的地方,應以考生的利益為依歸。
5. 評卷參考中，塗上陰影的部分代表可省略的步驟，有外框的部分代表運用不同方法的答案。所有分數答案必須化簡。

F.6 Mathematics 2023 Mock Exam Paper I

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SECTION A(1) (35 marks)

1.
$$\frac{(p^2q)^{-3}}{(p^3q^{-2})^{-4}}$$
$$= \frac{p^{-6}q^{-3}}{p^{-12}q^8}$$
 1M
$$= p^{-6-(-12)}q^{-3-8}$$
 1M
$$= p^6q^{-11}$$
$$= \frac{p^6}{q^{11}}$$
 1A

2.
$$\frac{7}{p} + \frac{2}{q} = \frac{3p}{4}$$
$$\frac{7q+2p}{pq} = \frac{3p}{4}$$
 1M
$$3p^2q = 4(7q+2p)$$
$$3p^2q = 28q + 8p$$
$$3p^2q - 28q = 8p$$
 1M
$$q(3p^2 - 28) = 8p$$
$$q = \frac{8p}{3p^2 - 28}$$
 1A

3.
$$\frac{x-3}{x+1} - \frac{x-5}{x-2}$$
$$= \frac{(x-3)(x-2) - (x-5)(x+1)}{(x+1)(x-2)}$$
 1M
$$= \frac{x^2 - 2x - 3x + 6 - x^2 - x + 5x + 5}{(x+1)(x-2)}$$
 1M
$$= \frac{-x+11}{(x+1)(x-2)}$$
 1A

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4. (a) $x^2 - 8x + 12$
 $= (x - 6)(x - 2)$ 1A
- (b) $xy - x^2 - 6y + 8x - 12$
 $= xy - 6y - (x^2 - 8x + 12)$
 $= y(x - 6) - (x - 6)(x - 2)$ 1M
 $= (x - 6)(y - x + 2)$ 1A
5. (a) $7x - 5 \leq 4x$ and 及 $15 + 4x \geq 2x$
 $3x \leq 5$ and 及 $2x \geq -15$ 1M
 $x \leq \frac{5}{3}$ and 及 $x \geq -\frac{15}{2}$ 1A
 $\therefore -\frac{15}{2} \leq x \leq \frac{5}{3}$ 1A
- (b) Number of integers 整數數量 = 9 1A
6. (a) $\angle OPQ = \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$ 1M + 1A
- (b) $R(2, 288^\circ)$ and $R(2, 348^\circ)$ 1A + 1A

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7. If the selling price decreases by \$300, the loss will increase by \$300.

若該衣櫃的售價減少 \$300，虧蝕會增加 \$300。

Let \$L\$ be the original loss by selling the wardrobe.

設售出該衣櫃所得的原來虧蝕為 \$L\$。

$$L \times 50\% = 300$$

1M

$$L \times 0.5 = 300$$

$$L = 600$$

∴ The original loss is \$600.

1A

原來的虧蝕是 \$600。

Let \$S\$ be the selling price of the wardrobe.

設該衣櫃的售價為 \$S\$。

Cost price 成本 = \$(S + 600)\$

1M

If the selling price increases by 20%,

若該衣櫃的售價增加 20%，

new selling price 新的售價 = \$S \times (1 + 20\%) = 1.2S\$

new loss 新的虧蝕 = \$[(S + 600) - 1.2S] = \text{\\$}[600 - 0.2S]\$

∴ $600 - 0.2S = 600 \times (1 - 60\%)$

1M

$$600 - 0.2S = 240$$

$$S = 1800$$

∴ The selling price of the wardrobe is \$1800.

1A

該衣櫃的售價是 \$1800。

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Alternative solution:

7. Let c and S be the cost and selling price of the wardrobe respectively.

設 c 及 S 分別為該衣櫃的成本和售價。

$$\text{Loss 虧蝕} = c - S$$

1M

Thus, we have

$$c - S(1 + 20\%) = (c - S)(1 - 60\%) \text{ and}$$

1M

$$c - (S - 300) = (c - S)(1 + 50\%)$$

1M

$$c - 1.2S = 0.4c - 0.4S$$

$$0.6c = 0.8S$$

$$c = \frac{4}{3}S$$

$$\therefore \frac{4}{3}S - (S - 300) = (\frac{4}{3}S - S)(1.5)$$

1M

$$S = 1800$$

\therefore The selling price of the wardrobe is \$1800.

1A

該衣櫃的售價是 \$1800。

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8. (a) $f(x) = k_1x^3 + k_2x$ where k_1 and k_2 are non-zero constants 其中 k_1 及 k_2 為非零常數 1M

$$f(2) = -30$$

$$k_1(2)^3 + k_2(2) = -30$$

$$8k_1 + 2k_2 = -30$$

$$4k_1 + k_2 = -15 \dots\dots(1)$$

$$f(5) = 240$$

$$k_1(5)^3 + k_2(5) = 240$$

$$125k_1 + 5k_2 = 240$$

$$25k_1 + k_2 = 48 \dots\dots(2)$$

1M

By solving (1) and (2), $k_1 = 3$, $k_2 = -27$

$$\therefore f(x) = 3x^3 - 27x$$

1A

(b) $f(x) = 0$

$$3x^3 - 27x = 0$$

1M

$$3x(x^2 - 9) = 0$$

$$x = 0 \quad \text{or} \quad x = \pm 3$$

1A

9. (a) $a = 7$

1A

$$b = 6$$

1A

$$c = 3$$

1A

- (b) The required probability 所求概率

$$= \frac{13 + 11 + 6}{40}$$

1M

$$= \frac{3}{4}$$

1A

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SECTION A(2) (35 marks)

10. (a) $GQ = GR$

$$12^2 + (2k + 12)^2 = (12k - 2k)^2$$

$$144 + 4k^2 + 48k + 144 = 100k^2$$

$$96k^2 - 48k - 288 = 0$$

$$(k - 2)(2k + 3) = 0$$

$$k = 2(\text{rejected}) \quad \text{or} \quad k = -\frac{3}{2}$$

1A

$$\therefore G(0, -3)$$

1A

(b) (i) Γ is the perpendicular bisector of RS .

1A

Γ 是 RS 的垂直平分線方程。

(ii) The required equation 所求方程

$$\frac{y+3}{x-0} = -1 \div \left(-\frac{1}{4}\right)$$

1M

$$y + 3 = 4x$$

$$4x - y - 3 = 0$$

1A

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11. (a) Refer to the notations in the figure.

如圖標明。

Let A_0 , A_1 and A_2 be the curved surface areas of cones VEF , VCD and VAB respectively.

設圓錐 VEF 、 VCD 和 VAB 的曲面面積分別是 A_0 、 A_1 和 A_2 。

Let x cm be the circumference of the water surface.

設水面的圓周是 x cm。

\therefore Cones VEF , VCD and VAB are similar.

圓錐 VEF 、 VCD 與 VAB 相似。

$$\therefore \frac{A_1}{A_0} = \left(\frac{x}{12}\right)^2 = \frac{x^2}{144} ; \frac{A_2}{A_0} = \left(\frac{20}{12}\right)^2 = \frac{25}{9}$$

1M+1M

$$\frac{A_2 - A_0}{A_1 - A_0} = \frac{256}{81}$$

$$\frac{\frac{A_2}{A_0} - \frac{A_0}{A_0}}{\frac{A_1}{A_0} - \frac{A_0}{A_0}} = \frac{256}{81}$$

$$\frac{\frac{25}{9} - 1}{\frac{x^2}{144} - 1} = \frac{256}{81}$$

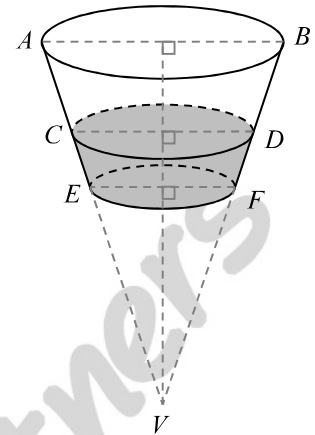
$$x^2 = 225$$

$$x = 15$$

\therefore The circumference of the water surface is 15 cm.

1A

水面的圓周為 15 cm。



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11. (b) Let $V \text{ cm}^3$ be the volume of the cone VEF .

設圓錐 VEF 的體積是 $V \text{ cm}^3$ 。

Volume of cone VAB

圓錐 VAB 的體積

$$= \left(\frac{20}{12}\right)^3 \times V \text{ cm}^3 = \frac{8\,000V}{1\,728} \text{ cm}^3 \quad 1A$$

Volume of cone VCD

圓錐 VCD 的體積

$$= \left(\frac{15}{12}\right)^3 \times V \text{ cm}^3 = \frac{3\,375V}{1\,728} \text{ cm}^3 \quad 1A$$

\therefore $\frac{\text{Capacity of the cup 杯子的容量}}{\text{Volume of water 水的體積}}$

$$= \frac{\frac{8\,000V}{1\,728} - V}{\frac{3\,375V}{1\,728} - V} = \frac{\frac{6\,272V}{1\,728}}{\frac{1\,647V}{1\,728}} = 3.81, \text{ cor. to 3 sig. fig.} \quad 1M$$

$$\therefore \left(\frac{16}{9}\right)^3 = 5.618\dots \neq 3.81$$

\therefore The claim is disagreed. 1A [f.t]

不同意該宣稱。

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12. (a) \therefore median 中位數 = 19.5

$$\therefore 3 + a + 4 = b + 4$$

$$3 + a = b$$

1M

$$\therefore b > 10$$

$$\therefore 3 + a > 10$$

$$a > 7$$

$$\therefore 7 < a < 10$$

$$\therefore a = 8, b = 11 \text{ or } a = 9, b = 12$$

1A + 1A

(b) (i) Greatest possible median 最大中位數 = 20

1A

(ii) By (a), there are two cases.

Case 1: $a = 8, b = 11$

Mean 平均數

$$= \frac{17 \times 4 + 18 \times 9 + 19 \times 5 + 20 \times 11 + 21 \times 4}{4 + 9 + 5 + 11 + 4}$$

$$= 19.06060606$$

1M

Case 2: $a = 9, b = 12$

Mean 平均數

$$= \frac{17 \times 4 + 18 \times 10 + 19 \times 5 + 20 \times 12 + 21 \times 4}{4 + 10 + 5 + 12 + 4}$$

$$= 19.05714286$$

1M

\therefore Least possible mean 最小平均數

$$= 19.05714286$$

$$= 19.1, \text{ cor. to 3 sig. fig.}$$

1A[ft.]

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13. (a) $\because BC = CQ$ (given)(已知)
- $\therefore \angle CBQ = \angle CQB$ (base \angle s, isos. Δ)(等腰 Δ 底角)
- $$\angle CBQ = \frac{180^\circ - 90^\circ}{2} = 45^\circ$$
- 1A
- (b) Let $\angle DBQ = x$.
- $$\angle CBN = x + \angle CBQ = x + 45^\circ$$
- 1M
- $\therefore CN = BN$
- $\therefore \angle BCN = \angle CBN = x + 45^\circ$ (base \angle s, isos. Δ)(等腰 Δ 底角) 1A
- $$\angle PDN + \angle DPN = \angle CND$$
- (ext.
- \angle
- of
- Δ
-)(
- Δ
- 外角)
- $$\angle BCN + \angle CBN = \angle CND$$
- (ext.
- \angle
- of
- Δ
-)(
- Δ
- 外角)
- $\therefore \angle PDN + \angle DPN = \angle BCN + \angle CBN$ 1M
- $$\angle PDN + 90^\circ = x + 45^\circ + x + 45^\circ$$
- $$\angle PDN = 2x$$
- $\therefore \angle PDN = 2\angle DBQ$ 1A
- (c) $\angle DRQ + \angle DBQ = \angle PDN$ (ext. \angle of Δ)(Δ 外角)
- $$\angle DRQ + \angle DBQ = 2\angle DBQ$$
- $$\angle DRQ = \angle DBQ$$
- $\therefore DR = DB$ (sides opp. eq. \angle s)(等角對邊相等) 1M
- $\therefore DB = CE$
- $\therefore DR = CE$ 1A

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14. (a) $f(-2 + 1) = f(2 + 1)$

$$f(-1) = f(3) \quad 1M$$

$$[3(-1)^2 - 5(-1) + h][2(-1)^2 - 5(-1) + k] = [3(3)^2 - 5(3) + h][2(3)^2 - 5(3) + k]$$

$$(8 + h)(7 + k) = (12 + h)(3 + k)$$

$$56 + 7h + 8k + hk = 36 + 3h + 12k + hk$$

$$4h - 4k = -20$$

$$h - k = -5 \quad \dots\dots(1) \quad 1A$$

Consider the coefficient of x^2 ,

$$2h + 3k + 25 = 60 \quad 1M$$

$$2h + 3k = 35 \quad \dots\dots(2) \quad 1A$$

By solving (1) and (2), $h = 4$, $k = 9$ 1A

(b) $f(x) = (3x^2 - 5x + 4)(2x^2 - 5x + 9)$ 1M

$$(3x^2 - 5x + 4)(2x^2 - 5x + 9) = 0$$

$$3x^2 - 5x + 4 = 0 \quad \text{or} \quad 2x^2 - 5x + 9 = 0$$

Consider $3x^2 - 5x + 4 = 0$, discriminant 判別式 $(-5)^2 - 4(3)(4) = -23 < 0$ 1M

Consider $2x^2 - 5x + 9 = 0$, discriminant 判別式 $(-5)^2 - 4(2)(9) = -47 < 0$ 1M

\therefore The equation $f(x) = 0$ has no real roots.

方程 $f(x) = 0$ 沒有實根。 1A[f.t.]

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SECTION B (35 marks)

15. The required probability 所求概率

$$\begin{aligned} &= \frac{P_7^7 \times P_4^8}{P_{11}^{11}} && 1M + 1M \\ &= \frac{7}{33} && 1A \end{aligned}$$

16. (a) Let s be the standard deviation.

設 s 為標準差。

$$\frac{139-118}{s} = 1.5$$

$$s = 14$$

1A

$$\text{Standard score 標準分} = \frac{90-118}{14} = -2$$

1A

- (b) Scores of Michael = Mean of the scores = 118

小高的得分 = 平均得分

$$\therefore \text{New mean} = \text{Original mean}$$

新的平均值 = 原來的平均值

New standard deviation > Original standard deviation

新的標準差 > 原來的標準差

$$\therefore \text{Standard score of Kenny will be increased.}$$

1A[f.t.]

大強標準分將會增加。

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17. (a) First term 首項

$$= -34 - 4(11) \quad 1M$$

$$= -78 \quad 1A$$

(b) Let T_n be the n th term of the sequence.

設 T_n 為該數列的第 n 項。

$$T_n = -78 + 11(n-1)$$

$$T_n < 0$$

$$-78 + 11(n-1) < 0$$

$$-78 + 11n - 11 < 0$$

$$11n < 89$$

$$n < \frac{89}{11}$$

$$\therefore n = 8 \quad 1A$$

\therefore The required sum 所求之和

$$= \frac{8}{2} [2(-78) + 11(8-1)]$$

$$= -316 \quad 1A$$

F.6 Mathematics 2023 Mock Exam Paper I

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18. (a) $f(x) = -\frac{1}{2}x^2 + 2kx + (3k - 5)$

$$f(x) = -\frac{1}{2}(x^2 - 4kx) + (3k - 5) \quad 1M$$

$$f(x) = -\frac{1}{2}(x - 2k)^2 + 2k^2 + (3k - 5)$$

$$f(x) = -\frac{1}{2}(x - 2k)^2 + 2k^2 + 3k - 5$$

$$\therefore \text{Vertex 頂點} = (2k, 2k^2 + 3k - 5) \quad 1A$$

(b) The y -coordinate of the vertex = 0

$$2k^2 + 3k - 5 = 0 \quad 1M$$

$$(2k + 5)(k - 1) = 0$$

$$k = -\frac{5}{2} \text{ or } k = 1 \quad 1A$$

Alternative solution:

Discriminant 判別式 = 0

$$(2k)^2 - 4\left(-\frac{1}{2}\right)(3k - 5) = 0 \quad 1M$$

$$4k^2 + 6k - 10 = 0$$

$$k = -\frac{5}{2} \text{ or } k = 1 \quad 1A$$

(c) $-2x^2 - 4kx + (3k - 5) = -\frac{1}{2}(-2x)^2 + 2k(-2x) + (3k - 5)$

$$= f(-2x) \quad 1A$$

$\therefore f(x)$ is contracting $\frac{1}{2}$ time along the x -axis and then reflected about y -axis. 1A

$f(x)$ 沿 x -軸方向縮小至 $\frac{1}{2}$ 倍及沿 y -軸反射。

OR $f(x)$ is reflected about y -axis and then contracting $\frac{1}{2}$ time along the x -axis. 1A

$f(x)$ 沿 y -軸反射及沿 x -軸方向縮小至 $\frac{1}{2}$ 倍。

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19. (a) Join BD .

$$\triangle ABD \cong \triangle CBD$$

$$\angle ABD = \angle CBD = 45^\circ; \angle ADB = \angle CDB = 60^\circ$$

$$\frac{AB}{\sin \angle ADB} = \frac{AD}{\sin \angle ABD}$$

$$\frac{AB}{\sin 45^\circ} = \frac{AD}{\sin 60^\circ}$$

$$AB = \frac{8\sqrt{6}}{3} \text{ cm or } 6.531972647 \text{ cm}$$

1A

Let F be the foot of perpendicular of E on CD .

設 F 為由 E 到 CD 的垂足。

$$CF = 8 \cos 60^\circ = 4 \text{ cm}$$

$$\angle BCD = 75^\circ$$

$$BF^2 = \sqrt{\left(\frac{8\sqrt{6}}{3}\right)^2 + 4^2 - 2 \times 4 \left(\frac{8\sqrt{6}}{3}\right) \cos 75^\circ} = 6.718770370 \text{ cm}$$

1M

$$EF = 8 \sin 60^\circ = 4\sqrt{3} \text{ cm or } 6.92820323 \text{ cm}$$

1M

$$BE = \sqrt{6.718770370^2 + (4\sqrt{3})^2} = 9.651003848 = 9.65 \text{ cm}$$

1A

(b) $AE = \sqrt{8^2 + 8^2} = 8\sqrt{2} \text{ cm}$

$$\text{Let } s = \frac{9.651003848 + 8\sqrt{2} + \frac{8\sqrt{6}}{3}}{2} = 13.74834250 \text{ cm}$$

Area 面積

$$= \sqrt{s(s - 9.651003848)(s - 8\sqrt{2})(s - \frac{8\sqrt{6}}{3})} = 31.45953756 \text{ cm}^2 > 30 \text{ cm}^2$$

1A

\therefore The claim is disagreed.

1A[f.t.]

該宣稱不正確。

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20. (a) Join OG .

連接 OG 。

Let $\angle ABO = a$

$$\angle AGO = 2\angle ABO = 2a \quad (\angle \text{ at centre} = 2 \times \angle \text{ at } \odot^{\text{c}}) \text{ (圓心角兩倍於圓周角)}$$

$$OG = AG \quad (\text{radii}) \text{ (半徑)}$$

$$\angle OAG = \frac{180^\circ - \angle AGO}{2} \quad (\angle \text{ sum of } \Delta) \text{ (}\Delta \text{內角和)}$$

$$= \frac{180^\circ - 2a}{2}$$

$$= 90^\circ - a$$

$$\angle OAG = \angle GAB = 90^\circ - a$$

$$\therefore \angle AOB = 180^\circ - \angle OAB - \angle ABO \quad (\angle \text{ sum of } \Delta) \text{ (}\Delta \text{內角和)}$$

$$= 180^\circ - 2(90^\circ - a) - a$$

$$= a$$

$$= \angle ABO$$

$$\therefore OA = AB \quad (\text{sides opp. equal } \angle\text{s}) \text{ (等角對邊相等)}$$

Marking Scheme:

Case 1: Any correct proof with correct reasons. 3M

Case 2: Any correct proof without reasons. 2M

Case 3: Incomplete proof with any one correct step and one correct reason. 1M

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Alternative solution:

Join OG and GB .

連接 OG 及 GB 。

Let $\angle OAG = b$

$$\angle GAB = \angle OAG = b$$

$$OG = AG = GB \quad (\text{radii})(\text{半徑})$$

$$\angle GOA = \angle OAG = b \text{ and } \angle GAB = \angle GBA = b \quad (\text{base } \angle, \text{isos } \Delta)(\text{等腰 } \Delta \text{ 底角})$$

$$AG = AG \quad (\text{common side})(\text{公共邊})$$

$$\therefore \triangle OAG \cong \triangle BAG \quad (\text{AAS})$$

$$OA = BA \quad (\text{corr. sides, } \cong \Delta\text{s})(\text{全等 } \Delta \text{ 對應邊})$$

Alternative solution:

Join OG and GB .

連接 OG 及 GB 。

Let $\angle OAG = b$

$$\angle GAB = \angle OAG = b$$

$$OG = AG = GB \quad (\text{radii})(\text{半徑})$$

$$\angle GOA = \angle OAG = b \text{ and } \angle GAB = \angle GBA = b \quad (\text{base } \angle, \text{isos } \Delta)(\text{等腰 } \Delta \text{ 底角})$$

Let $\angle BOG = c$

$$\angle OBG = \angle BOG = c \quad (\text{base } \angle, \text{isos } \Delta)(\text{等腰 } \Delta \text{ 底角})$$

$$\therefore \angle AOB = \angle AOG - \angle BOG = b - c$$

$$\angle ABO = \angle ABG - \angle OBG = b - c = \angle AOB$$

$$\therefore OA = AB \quad (\text{sides opp. equal } \angle\text{s})(\text{等角對邊相等})$$

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(b) (i) $OA = AB$

$$\sqrt{(x-0)^2 + (-1-0)^2} = \sqrt{(x+5)^2 + (-1+1)^2} \quad 1M$$

$$x = -\frac{12}{5}$$

$$\therefore A\left(-\frac{12}{5}, -1\right) \quad 1A$$

Let $C: x^2 + y^2 + Dx + Ey + F = 0$

$$\text{Put } (0, 0) \text{ into } C, (0)^2 + (0)^2 + D(0) + E(0) + F = 0 \quad \Rightarrow F = 0$$

$$\text{Put } (-5, -1) \text{ into } C, (-5)^2 + (-1)^2 + D(-5) + E(-1) = 0 \quad \Rightarrow 5D + E = 26 \dots\dots(1) \quad 1M$$

$$\text{Put } \left(-\frac{12}{5}, -1\right) \text{ into } C, \left(-\frac{12}{5}\right)^2 + (-1)^2 + D\left(-\frac{12}{5}\right) + E(-1) = 0 \quad \Rightarrow \frac{12}{5}D + E = \frac{169}{25} \dots\dots(2)$$

By solving (1) and (2), we have $D = \frac{37}{5}$, $E = -11$

$$\therefore C: x^2 + y^2 + \frac{37}{5}x - 11y = 0 \quad 1A$$

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(b) (ii) Let $L: y = 3x + c$

$$\begin{cases} x^2 + y^2 + \frac{37}{5}x - 11y = 0 \dots\dots(1) \\ y = 3x + c \dots\dots(2) \end{cases} \quad 1M$$

Put (1) into (2),

$$x^2 + (3x + c)^2 + \frac{37}{5}x - 11(3x + c) = 0$$

$$10x^2 + (6c - \frac{128}{5})x + (c^2 - 11c) = 0$$

$$50x^2 + (30c - 128)x + (5c^2 - 55c) = 0$$

$$\Delta = 0 \quad 1M$$

$$(30c - 128)^2 - 4(50)(5c^2 - 55c) = 0$$

$$c^2 - \frac{166}{5}c - \frac{4096}{25} = 0 \quad 1A$$

Let c_1 and c_2 be the roots of the above equation, where $c_1 > 0 > c_2$.

設 c_1 及 c_2 為上述方程的根，其中 $c_1 > 0 > c_2$ 。

$$Q(0, c_1), P(-\frac{c_1}{3}, 0), H(-\frac{c_2}{3}, 0), K(0, c_2)$$

$$\text{Area of } \triangle PQH = \frac{1}{2} \times (-\frac{c_2}{3} + \frac{c_1}{3}) \times c_1 = \frac{1}{6} c_1 (c_1 - c_2)$$

$$\text{Area of } \triangle PKH = \frac{1}{2} \times (-\frac{c_2}{3} + \frac{c_1}{3}) \times (-c_2) = \frac{1}{6} (-c_2)(c_1 - c_2)$$

Area of trapezium $PQHK$ 梯形 $PQHK$ 的面積

$$= \frac{1}{6} c_1 (c_1 - c_2) + \frac{1}{6} (-c_2)(c_1 - c_2) = \frac{1}{6} (c_1 - c_2)^2 \quad 1A$$

$$\text{Consider } c^2 - \frac{166}{5}c - \frac{4096}{25} = 0, \quad c_1 + c_2 = \frac{166}{5}, \quad c_1 c_2 = -\frac{4096}{25}$$

$$(c_1 - c_2)^2 = (c_1 + c_2)^2 - 4c_1 c_2 = (\frac{166}{5})^2 - 4(-\frac{4096}{25}) = \frac{8788}{5}$$

$$\therefore \text{Area of trapezium } PQHK = \frac{1}{6} \times \frac{8788}{5} = \frac{4394}{15} > 290$$

\therefore The claim is correct. 1A[f.t.]

該宣稱正確。