Paper 2:

1.	A	16.	C	31.	C
2.	D	17.	A	32.	D
3.	B	18.	A	33.	A
4.	C	19.	A	34.	A
5.	A	20.	D	35.	C
6.	A	21.	D	36.	D
7.	A	22.	C	37.	D
8.	C	23.	B	38.	C
9.	B	24.	D	39.	C
10.	B	25.	B	40.	B
11.	D	26.	D	41.	B
12.	C	27.	D	42.	B
13.	D	28.	B	43.	B
14.	C	29.	A	44.	B
15.	B	30.	A	45.	A

MC Solution:

1. A

$(-27)^{333} \times 2^{999} = (-3)^{999} \times 2^{999} = -(3 \times 2)^{999} = -6^{999}$

2. D

$$\frac{c-u}{d-u} = \frac{a}{b}$$

$$bc - bu = ad - au$$

$$au - bu = ad - bc$$

$$u(a-b) = ad - bc$$

$$u = \frac{ad-bc}{a-b}$$

3. B

$b^2(a+1) - (a+1) = (a+1)(b^2-1) = (a+1)(b+1)(b-1)$

4. C / D

$\sin 89^\circ = 0.999847695\ldots \approx 1.000$ (correct to 3 significant figures).

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5. A

$$\begin{cases} 5x - 3y = -1 \quad .....(1) \\ 21x - 3y = 15 \quad .....(2) \end{cases}$$

By solving (1) and (2), we have $x = 1$, $y = 2$

6. A

$p(2)^2 - (2) - 6 = 0$

$4p - 2 - 6 = 0$

$p = 2$

$\therefore \quad 2x^2 - x - 6 = (2x + 3)(x - 2)$

$\therefore \quad px^2 - x - 6$ is also divisible by $(2x + 3)$.

7. A

$-2 < -3x - 11 < 7$ or $-\frac{x+2}{2} \le 3$

$9 < -3x < 18$ or $x + 2 \ge -6$

$-3 > x > -6$ or $x \ge -8$

$\therefore \quad x \ge -8$

8. C

$2x^2 + (x + 2)k + 24 = 0$

$2x^2 + kx + 2k + 24 = 0$

$\Delta = 0$

$k^2 - 4(2)(2k + 24) = 0$

$k^2 - 16k - 192 = 0$

$(k + 8)(k - 24) = 0$

$k = -8$ or $k = 24$

9. B

$y = (ax + 1)^2 + k$

Coefficient of $x^2 = a^2 > 0$

$\therefore$ Opens upwards

Axis of symmetry = $-\frac{1}{a} > 0$

10. B

Let x be the cost of A.

$x(1 + 25\%)(30\%) = 600$

$x = 1600$

$\therefore$ The required profit

$= \$1600(25\%)$

$= \$400$

11. D

$p \cdot AB = q \cdot BC = r \cdot CA$

$\Rightarrow \frac{p}{6} = \frac{q}{4} = \frac{r}{3}$

$\therefore$ $p : q : r = 6 : 4 : 3$

$\therefore$ $p + q : q + r : r + p = (6 + 4) : (4 + 3) : (3 + 6) = 10 : 7 : 9$

12. C

Percentage change

$= [\frac{(1 + 20\%)^2}{1 - 20\%} - 1] \times 100\%$

$= 80\%$

13. D

Let $m : n$ be the required ratio.

$$\frac{25m+18n}{m+n}\times(1+30\%)=26$$

$$25m+18n=20(m+n)$$

$$25m+18n=20m+20n$$

$$5m=2n$$

$$\frac{m}{n}=\frac{2}{5}$$

$\therefore$ The required ratio = 2 : 5

14. C

1 st pattern: $2^2 + 1 = 5$

2 nd pattern: $3^2 + 1 = 10$

3 rd pattern: $4^2 + 1 = 17$

$\vdots$

7 rd pattern: $8^2 + 1 = 65$

15. B

$\angle RUV = 360^\circ - e$ ($\angle$ at a pt.)

$\angle UVR = 180^\circ - d$ (adj. $\angle$s on st. line)

$\angle SRQ = 360^\circ - e + 180^\circ - d$ (ext. $\angle$ of Δ)

$= 540^\circ - d - e$

$\angle QPS = \angle SRQ = 540^\circ - d - e$

$\angle TPS = 540^\circ - d - e - b$

$\angle TPS + \angle PTS + \angle TSP = 180^\circ$ ($\angle$ sum of Δ)

$540^\circ - d - e - b + c + a = 180^\circ$

$a + c + 360^\circ = b + d + e$

16. C

$\angle CAP = \angle BAG = 22.5^\circ$

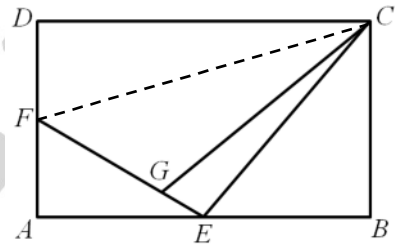
$\because$ $\Delta ABG \cong \Delta CBE$ (SAS)

$\therefore$ $\angle BCE = \angle BAG = 22.5^\circ$

$\therefore$ $\angle AEP = 180^\circ - 90^\circ - 22.5^\circ = 67.5^\circ$ ($\angle$ sum of Δ)

17. A

Join CF.



Area of ΔCEF

$= 10 \times 6 - \frac{1}{2}(5)(3) - \frac{1}{2}(5)(6) - \frac{1}{2}(10)(3)$

$= 22.5\,\text{cm}^2$

Area of ΔCEG

$= \frac{1}{4} \times 22.5\,\text{cm}^2$

$= 5.625\,\text{cm}^2$

18. A

Volume of container $= \frac{1}{2}(5)(5)\sin 60° \times 12 = 75\sqrt{3}\ \text{cm}^3$

Volume of water $= \frac{1}{2}(5)(5)\sin 60° \times 9 = 56.25\sqrt{3}\ \text{cm}^3$

Volume of space $= 75\sqrt{3} - 56.25\sqrt{3} = 18.75\sqrt{3}\ \text{cm}^3$

$\left(\frac{5\sin 60° - h}{5\sin 60°}\right)^2 = \frac{18.75\sqrt{3}}{75\sqrt{3}}$

$\left(\frac{5\sin 60° - h}{5\sin 60°}\right)^2 = \frac{1}{4}$

$\frac{5\sin 60° - h}{5\sin 60°} = \frac{1}{2}$

$h = 2.17$

19. A

Let $\angle COD = x$.

$$2\pi(OC + AC) \times \frac{x}{360} = 10 \text{.....(1)}$$

$$2\pi(OC) \times \frac{x}{360} = 7 \text{ .....(2)}$$

$$\frac{(1)}{(2)}: \quad \frac{OC + AC}{OC} = \frac{10}{7} \quad \Rightarrow \quad 7OC + 7AC = 10OC \quad \Rightarrow \quad AC = \frac{3}{7}OC$$

From (2), $\frac{\pi x}{360} = \frac{7}{2OC}$ .....(3)

Consider the area of the shaded region,

$$\pi(OC + AC)^2 \times \frac{x}{360} - \pi(OC)^2 \times \frac{x}{360} = 153$$

$$\pi(OC + \frac{3}{7}OC)^2 \times \frac{x}{360} - \pi(OC)^2 \times \frac{x}{360} = 153$$

$$\frac{\pi x}{360}[(\frac{10}{7}OC)^2 - (OC)^2] = 153$$

$$\frac{\pi x}{360} \times \frac{51}{49}OC^2 = 153$$

$$\frac{7}{2OC} \times \frac{51}{49}OC^2 = 153$$

$$\frac{51}{14}OC = 153$$

$$OC = 42 \text{ cm}$$

$\therefore$ I ✓

Area of the sector $COD = \pi(42)^2 \times \frac{x}{360} = (42)^2 \times \frac{7}{2 \times 42} = 147 \text{ cm}^2$

$\therefore$ II ✓

Perimeter of the sector $AOB = 10 + 42 + 42 + \frac{3}{7}(42) + \frac{3}{7}(42) = 130 \text{ cm}$

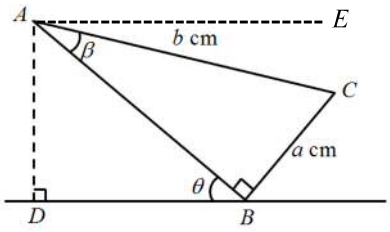
$\therefore$ III ×

20. D

Area of the shaded region $= \dfrac{(\frac{2}{3}+1)\times 1}{2} = \dfrac{5}{6}\text{ cm}^2$

21. D

Refer to the figure,



$\angle CAE = \theta - \beta$

$AD = (a\sin(180° - 90° - \theta) + b\sin(\theta - \beta))\text{ cm}$

$= (a\sin(90° - \theta) + b\sin(\theta - \beta))\text{ cm}$

$= (a\cos\theta + b\sin(\theta - \beta))\text{ cm}$

22. C

$\angle BOD = \angle ACD = 70°$ (properties of rhombus)

$\angle BAD = \dfrac{\angle BOD}{2} = \dfrac{70°}{2} = 35°$

$\angle ABE = \angle ACD = 70°$ (corr. $\angle$s, BO // CD)

$\angle AEB = 180° - 70° - 35° = 75°$ ($\angle$ sum of Δ)(Δ內角和)

23. B

Number of axes of reflectional symmetry = 4

24. D

Each interior angle of a regular 10-sided polygon

$= 180° - \frac{360°}{10} = 144°$

$\therefore$ I ✓

Number of axes of reflectional symmetry = 10

$\therefore$ II ✓

$\frac{144°}{36°} = 4$ times

$\therefore$ III ✓

25. B

Consider the y-intercept,

$$\frac{9}{n} = -\frac{15}{7}$$

$$n = -\frac{21}{5}$$

Consider the slope,

$$\frac{m}{-\frac{21}{5}} \times \frac{4}{7} = -1$$

$$m = \frac{147}{20}$$

26. D

Consider the x-intercept,

L_1 : x-intercept = B ; L_2 : x-intercept = D

$\therefore$ $B > D$

$\therefore$ III ✓

Consider the y-intercept,

L_1 : y-intercept = $\frac{B}{A} > 0$

$\because$ $B > 0$

$\therefore$ $A > 0$

$\therefore$ I ✓

Consider the slope,

L_1 : slope = $-\frac{1}{A}$; L_2 : slope = $-\frac{1}{C}$

$\because$ $-\frac{1}{A} \times -\frac{1}{C} = -1$

$\therefore$ $AC = -1$

$\therefore$ II ✓

27. D

C: $2x^2 + 2y^2 - 6x + 2y - 13 = 0 \Rightarrow x^2 + y^2 - 3x + y - \frac{13}{2} = 0$

Centre $= (\frac{3}{2}, -\frac{1}{2})$

$\therefore$ I ×

Radius $= \sqrt{(\frac{3}{2})^2 + (\frac{1}{2})^2 - (-\frac{13}{2})} = 3$

$AB = \sqrt{(1-2)^2 + (-2-1)^2} = \sqrt{10} > 3$

$\therefore$ II ✓

The equation of AB :

$$\frac{y+2}{x-1} = \frac{-2-1}{1-2}$$

$$y + 2 = 3x - 3$$

$$3x - y - 5 = 0$$

Put $(\frac{3}{2}, -\frac{1}{2})$ into $3x - y - 5$,

$$3(\frac{3}{2}) - (-\frac{1}{2}) - 5 = 0$$

$\therefore$ The centre of C lies on the straight line passing through A and B.

$\therefore$ III ✓

28. B

Prime number : 71, 73, 79

The required probability $= \frac{3}{10}$

29. A

Expected number of tokens obtained

$$=(\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{2}{4})\times 15+(1-\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{2}{4})\times 25$$

$$=\frac{3}{5}\times 15+\frac{2}{5}\times 25$$

$$=19$$

30. A

Boys:

Range = 80 – 52.5 = 27.5 kg

Minimum weight = 52.5 kg

Girls:

Range = 72.5 – 45 = 27.5 kg

Lower quartile = 52.5 kg

∴ I ✓

II ✓

∵ The number of boys and girls is unknown

∴ III ×

31. C

$3x(x+2)(x^2-x-6)=3x(x+2)^2(x-3)$

$6x^2(x+2)(x^2-9)=6x^2(x+2)(x+3)(x-3)$

$12(x-3)(x^3+8)=12(x-3)(x+2)(x^2-2x+4)$

∴ H.C.F. $=3(x-3)(x+2)$

32. D

$\log_8 a = \log_2 b$

$\frac{\log a}{\log 8} = \frac{\log b}{\log 2}$

$\frac{\log a}{3\log 2} = \frac{\log b}{\log 2}$

$\frac{\log a}{3} = \log b$

$\frac{\log a}{\log b} = 3$

$\therefore \quad \log a : \log b = 3 : 1$

33. A

11001000010001_2

$= 2^{13} + 2^{12} + 2^9 + 2^4 + 1$

$= 2^{13} + 2^9 + 2^{12} + 2^4 + 1$

$= 2^{13} + 2^9 + 4113$

34. A

$a_1 = 2$, $a_2 = 2$, $a_3 = 4$, $a_4 = 8$, $a_5 = 32$, $a_6 = 256$, $a_7 = 8192$, $a_8 = 2097152$, $a_9 = 17179869184$

Consider the unit digit,

the sequence are 2, 2, 4, 8, 2, 6, 2, 2, 4, …

$\therefore$ It will repeat itself for every 6 terms

$\because \quad 2015 = 6 \times 335 + 5$

$\therefore$ the unit digit of $a_{2015} = 2$

35. C

$h = -\frac{b}{2a}$; $k = \frac{4ac - b^2}{4a} = c - \frac{b^2}{4a}$

$k = c - \frac{b^2}{4a}$

$= c - \frac{b}{4} \cdot \frac{b}{a}$

$= c - \frac{b}{4} \cdot \frac{c}{b}$

$= \frac{3c}{4}$

$ck = \frac{3c^2}{4} > 0$

∴ I ✓

$h = -\frac{b}{2a}$

$ah = -\frac{b}{2}$

$abh = -\frac{b^2}{2} < 0$

∴ II ✓

$c - k = c - \frac{3c}{4} = \frac{c}{4}$

∵ $c > k$ if $c > 0$; $c < k$ if $c < 0$

∴ III ×

36. D

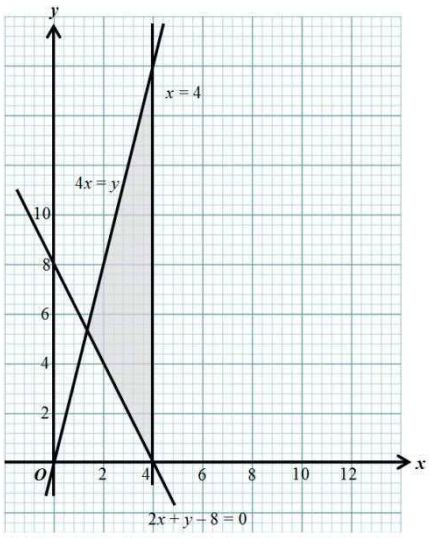
Let f(x, y) = x + 3y + 4

f(4, 0) = (4) + 3(0) + 4 = 8

f(4, 16) = (4) + 3(16) + 4 = 56

f($\frac{4}{3}$, $\frac{16}{3}$) = ($\frac{4}{3}$) + 3($\frac{16}{3}$) + 4 = $\frac{64}{3}$

∴ maximum value = 56



37. D

The graph of $y = g(x)$ is obtained by enlarged the graph of $y = f(x)$ along x-axis and then translated to the right.

$\therefore \quad g(x) = f(\frac{x}{2} - 1)$

38. C

In ΔACD, $AC = 5\cos x$, $DC = 5\sin x$

In ΔABC, $\cos\angle ABC = \dfrac{(AB)^2 + (BC)^2 - (AC)^2}{2(AB)(BC)}$

$$\cos x = \frac{(AB)^2 + 9 - 25\cos^2 x}{2(AB)(3)}$$

$$(AB)^2 - (6\cos x)AB + 9 - 25\cos^2 x = 0 \quad .....(1)$$

In ΔABD, $\cos\angle ABD = \dfrac{(AB)^2 + (BD)^2 - (AD)^2}{2(AB)(BD)}$

$$\cos x = \frac{(AB)^2 + 16 - 25}{2(AB)(4)}$$

$$(AB)^2 - (8\cos x)AB - 9 = 0 \quad .....(2)$$

(1) = (2),

$$(AB)^2 - (6\cos x)AB + 9 - 25\cos^2 x = (AB)^2 - (8\cos x)AB - 9$$

$$AB = \frac{25\cos^2 x - 18}{2\cos x} \quad .....(3)$$

Put (3) into (2),

$$(\frac{25\cos^2 x - 18}{2\cos x})^2 - (8\cos x)(\frac{25\cos^2 x - 18}{2\cos x}) - 9 = 0$$

$$225\cos^4 x - 648\cos^2 x + 324 = 0$$

$$\cos^2 x = 0.64401005 \quad \text{or} \quad \cos^2 x = 2.23598995 \,(\text{rejected})$$

$$\cos x = 0.802502367 \quad \text{or} \quad \cos x = -0.802502367 \,(\text{rejected})$$

$$x = 36.6302703°$$

$$\approx 37°$$

39. C

Join OG, OH and OD.

$\angle OGA = 90^\circ$ (line joining centre to mid-pt. of chord $\perp$ chord)(圓心至弦中點的連線⊥弦)

$\angle OHC = 90^\circ$ (line joining centre to mid-pt. of chord $\perp$ chord)(圓心至弦中點的連線⊥弦)

$\angle ODA = \angle ODC = 90^\circ$ (tangent properties)(切線性質)

$\therefore$ $OGDA$, $OHCD$ and $OGBH$ are cyclic quadrilateral. (opp. $\angle$s, supp.)(對角互補)

$\angle OAG = \angle ODG$ ($\angle$s in the same segment)(同弓形內的圓周角)

$\angle OCH = \angle ODH$ ($\angle$s in the same segment)(同弓形內的圓周角)

$\angle GOA = 180^\circ - 90^\circ - \angle OAG = 90^\circ - \angle OAG$ ($\angle$ sum of Δ)(Δ內角和)

$\angle HOC = 180^\circ - 90^\circ - \angle OCH = 90^\circ - \angle OCH$ ($\angle$ sum of Δ)(Δ內角和)

$\angle GOH = 180^\circ - x$ (opp. $\angle$s, cyclic quad.)(圓內接四邊形對角)

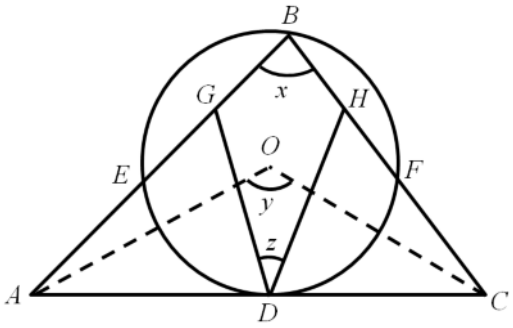
$\angle GOH + \angle HOC + \angle GOH + \angle AOC = 360^\circ$ ($\angle$ sum of Δ)(Δ內角和)

$180^\circ - x + 90^\circ - \angle OAG + 90^\circ - \angle OCH + y = 360^\circ$

$-x + y - (\angle OAG + \angle OCH) = 0$

$-x + y - z = 0$

$z = y - x$



40. B

Join BC.

$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)(半圓上的圓周角)

$\angle ACB = \angle ABD = 60^\circ$ ($\angle$ in alt segment)(交錯弓形的圓周角)

$\angle BAC = 180^\circ - 90^\circ - 60^\circ = 30^\circ$ ($\angle$ sum of Δ)(Δ內角和)

$\angle CBE = \angle BAC = 30^\circ$ ($\angle$ in alt segment)(交錯弓形的圓周角)

$\angle AEB = 180^\circ - 90^\circ - 30^\circ - 30^\circ = 30^\circ$ ($\angle$ sum of Δ)(Δ內角和)

$\angle FEA = 15^\circ$

$\angle BFE = 15^\circ + 30^\circ = 45^\circ$

41. B

Let m and n be the number of man and woman respectively.

$C_2^m = 66$; $C_2^n = 28$

$\frac{m(m-1)}{2} = 66$; $\frac{n(n-1)}{2} = 28$

$m^2 - m - 132 = 0$; $n^2 - n - 56 = 0$

$m = 12$ or $m = -11$ (rejected) ; $n = 8$ or $n = -7$ (rejected)

$\therefore$ Number of and shakes took place between a man and a woman

$= C_1^{12} \times C_1^8$

$= 96$

42. B

The required probability

$= \frac{4}{3!}$

$= \frac{2}{3}$

43. B

$$\begin{cases} x^2 + y^2 + x + 4y - 2 = 0 \text{.....(1)} \\ 4x + 3y + 8 = 0 \text{ .....(2)} \end{cases}$$

From (2), $y = \dfrac{-4x-8}{3}$ ……(3)

$$x^2 + (\frac{-4x-8}{3})^2 + x + 4(\frac{-4x-8}{3}) - 2 = 0$$

$$x^2 + \frac{16x^2 + 64x + 64}{9} + x - \frac{16x + 32}{3} - 2 = 0$$

$$9x^2 + 16x^2 + 64x + 64 + 9x - 48x - 96 - 18 = 0$$

$$25x^2 + 25x - 50 = 0$$

$$x^2 + x - 2 = 0$$

$$x = 1 \text{ or } x = -2$$

Put x = 1 into (3), $y = -4$

Put x = –2 into (3), $y = 0$

∴ The coordinates of the points A and B are (–2, 0) and (1, –4) respectively.

∴ I ✓

Radius = $\sqrt{(\frac{1}{2})^2 + 2^2 - (-2)} = \frac{5}{2}$

∴ Diameter = 5

∴ II ×

$G(-\frac{1}{2}, -2)$

Slope of AB = $\dfrac{-4-0}{1-(-2)} = -\dfrac{4}{3}$

Slope of AG = $\dfrac{-2-0}{-\frac{1}{2}-(-2)} = -\dfrac{4}{3}$

∴ The points A, B and G are collinear.

∴ III ✓

44. B

Let n be the number of students in the group and let $x_1, x_2, \ldots, x_n$ be the scores of the students, write $\mu_0 = \text{mean}$;

$\sigma_0 = \text{standard deviation of the scores originally.}$

Suppose x = Mary's score, $z = \dfrac{x-\mu_0}{\sigma_0}$

After adding the score of one more students mean $\mu_1 = \mu_0$;

Standard deviation σ_1

$$= \sqrt{\frac{(x_1-\mu_1)^2+(x_2-\mu_1)^2+\ldots+(x_n-\mu_1)^2+(\mu_0-\mu_1)^2}{n+1}}$$

$$= \sqrt{\frac{(x_1-\mu_1)^2+(x_2-\mu_1)^2+\ldots+(x_n-\mu_1)^2}{n+1}}$$

$$= \sqrt{\frac{n\sigma_0^{\,2}}{n+1}}, \text{ where } \sigma_0 = \sqrt{\frac{(x_1-\mu_0)^2+(x_2-\mu_0)^2+\ldots+(x_n-\mu_0)^2}{n}}$$

$$= \sqrt{\frac{n}{n+1}}\sigma_0$$

Current standard score of Mary

$$z' = \frac{x-\mu_1}{\sigma_1}$$

$$= \frac{x-\mu_0}{\sqrt{\frac{n}{n+1}}\sigma_0}$$

$$= \sqrt{\frac{n}{n+1}} \cdot \frac{x-\mu_0}{\sigma_0}$$

$$= \sqrt{\frac{n}{n+1}} \cdot z$$

$\therefore$ $\dfrac{7\sqrt{3}}{12} = \sqrt{\dfrac{n}{n+1}}$

$$\frac{147}{144} = \frac{n}{n+1}$$

$$144n = 147n + 147$$

$$n = 48$$

45. A

For I,

$$m = \frac{a+b+c+d+e}{5} \Rightarrow a+b+c+d+e = 5m$$

Mean of number in Set B

$$= \frac{a+b+c+d+e+m}{5}$$

$$= \frac{5m+m}{6}$$

$$= m$$

$\therefore$ I ✓

For II & III,

Suppose A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 3, 4, 5}.

Inter-quartile range of numbers in Set A = 4.5 – 1.5 = 3

Inter-quartile range of numbers in Set B = 4 – 2 = 2

$\therefore$ II ×

Variance of numbers in Set A = 2

Variance of numbers in Set B $= \frac{5}{3}$

$\therefore$ III ×