F.6 Mathematics 2024 Mock Exam Paper I & II

Kit Lee & his partners

Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Paper 1

Solution	Marks	Remarks
1. $\dfrac{(x^2y^{-3})^9}{(x^3y)^{-4}}$		
$=\dfrac{x^{18}y^{-27}}{x^{-12}y^{-4}}$	1M	for $(a^h)^k = a^{hk}$ or $(ab)^l = a^lb^l$
$=x^{18-(-12)}y^{-27-(-4)}$	1M	for $\dfrac{c^p}{c^q}=c^{p-q}$ or $d^{-r}=\dfrac{1}{d^r}$
$=x^{30}y^{-23}$		
$=\dfrac{x^{30}}{y^{23}}$	1A	
	--------(3)	
2. $\dfrac{p(1+q)}{5}=\dfrac{qr}{2}+1$		
$10\times\dfrac{p(1+q)}{5}=10\times(\dfrac{qr}{2}+1)$		
$2p(1+q)=5qr+10$		
$2p+2pq=5qr+10$	1M	
$2pq-5qr=10-2p$	1M	for putting q on one side
$q(2p-5r)=10-2p$		
$q=\dfrac{10-2p}{2p-5r}$	1A	or equivalent
	--------(3)	
3. (a) $2x^2+3xy-35y^2$		
$=(2x-7y)(x+5y)$	1A	or equivalent
(b) $4x-14y-2x^2-3xy+35y^2$		
$=4x-14y-(2x^2+3xy-35y^2)$		
$=2(2x-7y)-(2x-7y)(x+5y)$	1M	for using the result of (a)
$=(2x-7y)[2-(x+5y)]$		
$=(2x-7y)(2-x-5y)$	1A	or equivalent
	--------(4)	

Solution	Marks	Remarks
4. (a) $\frac{4(x-1)}{5}+10>7(x-4)$		
$4(x-1)+50>35(x-4)$		
$4x-4+50>35x-140$		
$4x-35x>-140+4-50$	1M	for putting x on one side
$-31x>-186$		
$x<6$	1A	
$x+3\geq 0$		
$x\geq -3$		
Thus, we have $-3\leq x<6$.	1A	
(b) 5	1A	
	--------(4)	
5. Let x and y be the number of candies in the box and the number of students in the class respectively.		
$\begin{cases} x-30=4y \\ x=6(y-4) \end{cases}$	1A + 1A	
So, we have $6(y-4)-30=4y$	1M	for getting a linear equation in one unknown
Solving, we have $y=27$ and $x=138$		
Thus, the number of candies in the box is 138.	1A	
	--------(4)	

Solution	Marks	Remarks
6. Let $\$x$ be the marked price of the robot.		
The cost of the robot		
$= \$(x - 100)$	1M	
The selling price of the robot		
$= (70\%)x$	1M	
$= \$0.7x$		
$0.7x = (x - 100)(1 - 20\%)$	1M	
$0.7x = 0.8x - 80$		
$x = 800$	1A	
Thus, the marked price of the robot is \$800.		
Let $\$c$ be the cost price of the robot.		
The marked price of the robot		
$= \$(c + 100)$	1M	
The selling price of the robot		
$= (c + 100)(70\%)$		
$= \$(0.7c + 70)$	1M	
$0.7c + 70 = (1 - 20\%)c$		
$0.7c + 70 = 0.8c$	1M	
$c = 700$		
Thus, the marked price of the robot is \$800.	1A	
	--------(4)	

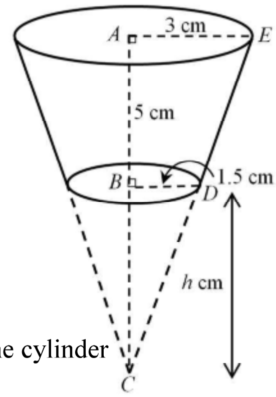
Solution	Marks	Remarks
7. (a) The coordinates of P are $(-3, -4)$. The coordinates of Q are $(3, 4)$.	1A	for both correct
(b) $P'(-4, 3)$	1M	
The slope of OP' $=\frac{3-0}{-4-0}=-\frac{3}{4}$	1M	either one
$Q'(3, -4)$ The slope of OQ' $=\frac{-4-0}{3-0}=-\frac{4}{3}$		
Note that slope of $OP' \neq$ slope of OQ'		
Thus, O, P' and Q' are not collinear.	1A	f.t.
	--------(4)	
8. (a) Consider ΔABE and ΔFCE.		
$AE = FE$ [property of equilateral triangle]		
$BE = CE$ [property of equilateral triangle]		
$\angle BEA = \angle BEC - \angle AEC$ $= 60^\circ - \angle AEC$ $= \angle AEF - \angle AEC$ $= \angle CEF$		
$\Delta ABE \cong \Delta FCE$ (SAS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	for $\Delta ABE \cong \Delta FCE$ or $\Delta FDA \cong \Delta FCE$
Case 2 Any correct proof without reasons.	1	
Similarly, $\Delta FDA \cong \Delta FCE$ (SAS)		
Thus, $\Delta ABE \cong \Delta FDA$.		
$AB = FD$ [corr. sides, $\cong$ Δs]	1A	
$= DC$ [property of equilateral triangle]		
	--------(3)	

Solution	Marks	Remarks
8. (b) Consider ΔFCE.		
$\angle ECF + \angle CEF + \angle CFE = 180^\circ$		
$140^\circ + \angle CEF + \angle CFE = 180^\circ$		
$\angle CEF + \angle CFE = 40^\circ$	1M	
Note that $\angle DAF = \angle CEF$ and $\angle BAE = \angle CFE$		
$\angle BAD = \angle DAF + \angle BAE + \angle EAF$		
$= \angle CEF + \angle CFE + 60^\circ$		
$= 40^\circ + 60^\circ$		
$= 100^\circ$	1A	
	--------(2)	
9. (a) The least possible value of the inter-quartile range = 0	1A	
The greatest possible value of the inter-quartile range = 3	1A	
(b) $h + 5 + 8 > k$	1M	
$2k + 13 > k$		
$k > -13$		
$h + 5 < 8 + k$		
$2k + 5 < 8 + k$	1M	
$k < 3$		
Thus, $0 < k < 3$		
There are 2 possible values of k.	1A	
	--------(5)	

Solution	Marks	Remarks
10. (a) Let $f(x) = ax^2 + b$, where a and b are non-zero constants.	1M	
So, we have $196a + b = 79$ and $441a + b = 154$.	1M	for either substitution
Solving, we have $a = \frac{15}{49}$ and $b = 19$.		
Hence, we have $f(x) = \frac{15}{49}x^2 + 19$.		
Thus, we have $f(7) = 34$.	1A	
	---------(3)	
(b) By (a), we have $p = 19$ and $q = 34$.	1M	for either one
So, we have $PR = 15$ and $QR = 7$		
Note that $\angle PRQ = 90^\circ$		
The area of ΔPQR		
$= \frac{15 \times 7}{2}$	1M	
$= 52.5$ sq. units	1A	
	---------(3)	
11. (a) $99 - (50 + a) = 46$		
$a = 3$	1A	
$\frac{1512 + b}{20} = 76$		
$b = 8$	1A	
	---------(2)	
(b) The required probability $= \frac{10}{20}$	1M	for denominator
$= \frac{1}{2}$	1A	
	---------(2)	
(c) The required probability $= \frac{\frac{3}{20} \times \frac{2}{19}}{\frac{10}{20} \times \frac{9}{19}}$	1M	for denominator
$= \frac{1}{15}$	1A	
	---------(2)	

Solution	Marks	Remarks
12. (a) Let $f(x) = (x-2)^2 q(x) + x - 8 - k$, where q(x) is a binomial.	1M	either one
$f(2) = (2-2)^2 q(2) + (2) - 8 - k = -6 - k$		
Let $f(x) = (x-2)(2x+1)p(x) - \frac{3}{2}kx + 6$, where p(x) is a binomial.		
$f(2) = (2-2)[2(2)+1]p(2) - \frac{3}{2}k(2) + 6 = 6 - 3k$		
Thus, we have		
$-6 - k = 6 - 3k$	1M	
$k = 6$	1A	
	--------(3)	
(b) $f(x) = (x-2)(2x+1)p(x) - 9x + 6$ (by (a))		
$f(-\frac{1}{2}) = [(-\frac{1}{2}) - 2][2(-\frac{1}{2}) + 1]p(-\frac{1}{2}) - 9(-\frac{1}{2}) + 6$		
$f(-\frac{1}{2}) = \frac{21}{2}$		
Let $f(x) = (x-2)^2(2x+a) + x - 14$, where a is a constant.		
$f(-\frac{1}{2}) = [(-\frac{1}{2}) - 2]^2[2(-\frac{1}{2}) + a] + (-\frac{1}{2}) - 14 = \frac{21}{2}$		
$a = 5$	1A	
Thus, $f(x) = (x-2)^2(2x+5) + x - 14$		
$(x-2)^2(2x+5) + x - 14 = -6x$		
$(x-2)^2(2x+5) + 7x - 14 = 0$		
$(x-2)^2(2x+5) + 7(x-2) = 0$		
$(x-2)[(x-2)(2x+5) + 7] = 0$		
$(x-2)(2x^2 + x - 3) = 0$	1M	
$(x-2)(x-1)(2x+3) = 0$		
$x = 2$, $x = 1$ or $x = -\frac{3}{2}$	1M	
Note that both $x = 2$, $x = 1$ and $x = -\frac{3}{2}$ are rational roots of $f(x) = -6x$.		
Thus, the claim is correct.	1A	f.t.
	--------(4)	

Solution	Marks	Remarks
13. (a) Consider the frustum,		
Since, $\Delta ACE \sim \Delta BCD$ (AAA)		
So, $\frac{h}{h+5} = \frac{1.5}{3}$ [corr. sides, ~Δs]		
$h = 5$	1M	
The required capacity		
= capacity of the frustum + capacity of the cylinder		
$= [\frac{1}{3}\pi(3)^2(5+5) - \frac{1}{3}\pi(1.5)^2(5)] + \pi(1.5)^2(3)$	1M	
$= 33\pi \text{ cm}^3$	1A	
	--------(3)	
(b) The volume of the ice-cream		
$= \frac{1}{2} \times \frac{4}{3}\pi(2.4)^3$	1M	
$= 9.216\pi \text{ cm}^3$	1M	
The volume of the milkshake		
$= 9.216\pi + 15$		
$= 43.9529179 \text{ cm}^3$	1M	
$< 33\pi \text{ cm}^3$		
So, the 'milkshake' will not overflow.	1A	f.t.
	--------(4)	



Solution	Marks	Remarks
14. (a) (i) Γ is the perpendicular bisector of AB.	1A	
(ii) Let (x, y) be the coordinates of P.		
$\sqrt{(x-8)^2+(y+6)^2} = \sqrt{(x-16)^2+(y-6)^2}$	1M	
$2x + 3y - 24 = 0$	1A	or equivalent
Thus, the equation of Γ is $2x + 3y - 24 = 0$.		
	--------(3)	
(b) (i) The coordinates of the mid-point of $AB = (12, 0)$		
Let h be the x-coordinate of G.		
By (a)(ii), we have $2h + 3k - 24 = 0$		
So, we have $h = \frac{24-3k}{2}$	1M	
The equation of C is		
$(x-h)^2+(y-k)^2 = (12-h)^2+(0-k)^2$		
$x^2+y^2-2(\frac{24-3k}{2})x-2ky+24(\frac{24-3k}{2})-144=0$	1M	
$x^2+y^2-(24-3k)x-2ky+144-36k=0$	1	
(ii) Since, C passes through point $D(7, 5)$, we have		
$(7)^2 + (5)^2 - (24 - 3k)(7) - 2k(5) + 144 - 36k = 0$		
$k = 2$		
Thus, the coordinates of G are $(9, 2)$.	1A	
Radius of the smallest circle which passes through B and G		
$= \frac{1}{2}\sqrt{(16-9)^2+(6-2)^2}$		
$= \frac{\sqrt{65}}{2}$		
The required area		
$= \pi(\frac{\sqrt{65}}{2})^2$	1M	
$= \frac{65}{4}\pi$	1A	
	--------(6)	

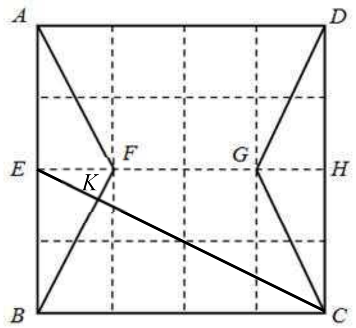
Solution	Marks	Remarks
15. (a) The required number		
$= P_{13}^{13}$		
$= 6227020800$	1A	
	--------(1)	
(b) The required probability		
$= \frac{9!C_4^{10}4!}{6227020800}$	1M+1M	1M for denominator + 1M for 9!4!
$= \frac{1828915200}{6227020800}$		
$= \frac{42}{143}$	1A	r.t. 0.294
	--------(3)	

Solution	Marks	Remarks
Notice that the question in Q16 is: Suppose a, 27, b is a geometric sequence, where $b < 1 < a$. *Appropriate marks will be awarded based on the answers given by the candidates.		
16. (a) $\frac{b}{27}=\frac{27}{a}$		
$ab = 729$		
$\log_3 ab = \log_3 729$		
$\log_3 a + \log_3 b = 6$	1M	
$\log_3 b = 6 - \log_3 a$	1A	
	--------(2)	
(b) $\log_{27} ab - \log_a 27b = \log_a 27b - \log_b 27a$	1M	
$\log_{27} ab = 2\log_a 27b - \log_b 27a$		
$\frac{\log_3 ab}{\log_3 27} = \frac{2\log_3 27b}{\log_3 a} - \frac{\log_3 27a}{\log_3 b}$		
$\frac{\log_3 a + \log_3 b}{3} = \frac{6+2\log_3 b}{\log_3 a} - \frac{3+\log_3 a}{\log_3 b}$	1M	for using the result of (a)
Let $u = \log_3 a$.		
$\frac{u+6-u}{3} = \frac{6+2(6-u)}{u} - \frac{3+u}{6-u}$ (by (a))	1M	
$u^2 - 15u + 36 = 0$		
$u = 12$ or $u = 3$ (rejected)		
$\log_3 a = 12$		
The common difference of the arithmetic sequence		
$= \log_a 27b - \log_b 27a$		
$= \frac{\log_3 27b}{\log_3 a} - \frac{\log_3 27a}{\log_3 b}$		
$= \frac{3+6-\log_3 a}{\log_3 a} - \frac{3+\log_3 a}{6-\log_3 a}$		
$= \frac{3+6-12}{12} - \frac{3+12}{6-12}$		
$= \frac{9}{4}$	1A	
	--------(4)	

Solution	Marks	Remarks
17. (a) f(x)		
$= -x^2 + 4kx - 13k^2 - 26$		
$= -[x^2 - 2(2kx) + (2k)^2 - (2k)^2] - 13k^2 - 26$	1M	
$= -(x - 2k)^2 - 9k^2 - 26$		
Thus, the coordinates of the vertex of the graph of y = f(x) are		
$(2k, -9k^2 - 26)$.	1A	
	--------(2)	
(b) g(x)		
$= k\mathrm{f}(x) + \dfrac{4-9k^2}{k}$	1M	
$= k[-(x-2k)^2 - 9k^2 - 26] + \dfrac{4-9k^2}{k}$		
$= -k(x-2k)^2 + \dfrac{4-35k^2-9k^4}{k}$		
Thus, the coordinates of the vertex of the graph of y = g(x) are		
$(2k,\ \dfrac{4-35k^2-9k^4}{k})$.		
Note that g(x) is always negative for all real values of x.		
Thus, the y-coordinate of the vertex of the graph of y = g(x) is always negative		
$\dfrac{4-35k^2-9k^4}{k} < 0$	1M	
$9k^4 + 35k^2 - 4 > 0$		
$(9k^2 - 1)(k^2 + 4) > 0$	1M	
$k^2 > \dfrac{1}{9}$ or $k^2 < -4$ (rejected)	1M	either one
$k > \dfrac{1}{3}$ or $k < -\dfrac{1}{3}$ (rejected)		
Thus, the range of values of k is $\dfrac{1}{3} < k < \dfrac{2}{3}$.	1A	
	--------(5)	

Solution	Marks	Remarks
18. (a) $L: y = \frac{485-8x}{15}$		
Putting $y = \frac{485-8x}{15}$ in $x^2 + y^2 - 4x - 24y + k = 0$, we have		
$x^2 + (\frac{485-8x}{15})^2 - 4x - 24(\frac{485-8x}{15}) + k = 0$	1M	
So, we have $289x^2 - 5780x + 60625 + 225k = 0$ ……(*)		
Since, L is the tangent to the circle C.		
Thus, $(-5780)^2 - 4(289)(60625 + 225k) = 0$	1M	
$k = -141$	1A	
Put $k = -141$ in (*), we have		
$289x^2 - 5780x + 28900 = 0$		
$x^2 - 20x + 100 = 0$		
$(x-10)^2 = 0$		
$x = 10$		
Put $x = 10$ in $y = \frac{485-8x}{15}$, we have $y = 27$.		
Thus, the coordinates of P are (10, 27).	1A	
	--------(4)	
(b) (i) $q = \frac{485-8(25)}{15} = 19$		
Thus, the coordinates of Q are (25, 19).		
The centre of the circumcircle = the mid-point of QG		
$= (\frac{27}{2}, \frac{31}{2})$	1M	
The radius of the circumcircle $= \sqrt{(\frac{27}{2} - 2)^2 + (\frac{31}{2} - 12)^2}$		
$= \sqrt{\frac{289}{2}}$	1M	
The required equation is		
$(x - \frac{27}{2})^2 + (y - \frac{31}{2})^2 = \frac{289}{2}$ (or $(2x-27)^2 + (2y-31)^2 = 578$)	1A	

Solution	Marks	Remarks
18. (b) (ii) The radius of the circumcircle $=\sqrt{\frac{289}{2}}$ (by (b)(i))		
The radius of $C=\sqrt{2^2+12^2-(-141)}=17$	1M	
The required ratio		
$=(\sqrt{\frac{289}{2}})^2:17^2$	1A	
$=1:2$		
Thus, the claim is correct.		
	--------(5)	
19. (a) (i) $EH=FG=20$ cm		
Add a mid-point M on AB.		
$BM^2+EM^2=BE^2$		
$BM=\sqrt{20^2-10^2}=10\sqrt{3}$ cm	1M	
$AB=20\sqrt{3}$		
$AB\approx 34.64101615$		
$AB\approx 34.6$ cm	1A	r.t. 34.6 cm
(ii) In ΔAEB, by cosine formula, we have		
$\cos\angle AEB=\frac{20^2+20^2-(20\sqrt{3})^2}{2(20)(20)}$	1M	
$\angle AEB=120°$	1A	
Thus, the required angle is 120°.		
(iii) In ΔBCE, $\angle ABC=90°$ & $\angle ABE=30°$		
Thus, $\angle EBC=90°-30°=60°$		
By cosine formula, we have		
$CE^2=20^2+40^2-2(20)(40)\cos 60°$	1M	
$CE\approx 34.64101615$ cm		
$CE\approx 34.6$ cm	1A	r.t. 34.6 cm
	--------(6)	

Solution	Marks	Remarks
19. (b) In figure 3(a), join CE and add a point K where K is the point of intersection of CE and BF.		
		
Since, $\Delta EFK \sim \Delta CBK$ (AAA) &		
$\Delta BFE \sim \Delta CEB$ [ratio of 2 sides, inc. $\angle$]		
Thus, we have $\Delta EFK \sim \Delta CBK \sim \Delta BFE$ (AAA)		
$\angle EKF = \angle CKB = \angle BEF = 90°$ and $EK : CK = EF : CB = 1 : 4$	1M	
In ΔCEB, $CE^2 = 40^2 + 20^2$		
$CE = 20\sqrt{5} \approx 44.72135955$ cm	1M	f.t.
Thus, we have		
$EK = 4\sqrt{5} \approx 8.94427191$ cm and $CK = 16\sqrt{5} \approx 35.77708764$ cm		
In figure (b), consider ΔCEK,		
$\cos \angle CKE = \dfrac{(4\sqrt{5})^2 + (16\sqrt{5})^2 - (20\sqrt{3})^2}{2(4\sqrt{5})(16\sqrt{5})}$		
$\angle CKE \approx 75.5° > 75°$	1A	
So, the angle between the planes BEF and $BCGF$ is greater 75°.		
Thus, the claim is correct.	--------(3)	

F.6 Mathematics 2024 Mock Exam Paper I & II

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Paper 2

Question No.	Key	Question No.	Key
1.	B	26.	A
2.	B	27.	C
3.	B	28.	B
4.	A	29.	D
5.	D	30.	A
6.	C	31.	D
7.	C	32.	C
8.	D	33.	A
9.	C	34.	A
10.	D	35.	A
11.	A	36.	B
12.	B	37.	A
13.	A	38.	D
14.	D	39.	C
15.	C	40.	C
16.	D	41.	A
17.	B	42.	C
18.	B	43.	C
19.	A	44.	D
20.	D	45.	B
21.	A		
22.	D		
23.	C		
24.	B		
25.	B		

1. B

$$\frac{p}{2-p}=\frac{q}{4+q}$$

$$p(4+q)=q(2-p)$$

$$4p+pq=2q-pq$$

$$4p+2pq=2q$$

$$2p+pq=q$$

$$p(2+q)=q$$

$$p=\frac{q}{2+q}$$

2. B

$$\frac{4}{3x+2}+\frac{3}{2-3x}=\frac{4}{3x+2}-\frac{3}{3x-2}$$

$$=\frac{4(3x-2)-3(3x+2)}{(3x+2)(3x-2)}$$

$$=\frac{12x-8-9x-6}{9x^2-4}$$

$$=\frac{3x-14}{9x^2-4}$$

3. B

$$\frac{7^y\cdot(7^y)^y}{7^{y+2}}=\frac{7^y\cdot 7^{y^2}}{7^{y+2}}$$

$$=7^{y+y^2-(y+2)}$$

$$=7^{y^2-2}$$

4. A

$$a^2-9b^2-8a+16=a^2-8a+16-9b^2$$

$$=(a-4)^2-9b^2$$

$$=[(a-4)-3b][(a-4)+3b]$$

$$=(a-4-3b)(a-4+3b)$$

$$=(a-3b-4)(a+3b-4)$$

5. D

$(ax-2)(2x+3)+3 \equiv bx^2+(a+2)x-3$

$2ax^2+3ax-4x-6+3 \equiv bx^2+(a+2)x-3$

$2ax^2+(3a-4)x-3 \equiv bx^2+(a+2)x-3$

By comparing the coefficient of x, we have

$3a-4=a+2$

$2a=6$

$a=3$

By comparing the coefficient of x^2, we have

$b=2(3)=6$

6. C

$2(15-x) \le 2x+6$

$30-2x \le 2x+6$

$-4x \le -24$

$x \ge 6$

and $2x \le x+6$

$x \le 6$

Thus, $x=6$

7. C

8. D

$f(x)=2x^2+1$

$f(x-1)=2(x-1)^2+1$

$=2(x^2-2x+1)+1$

$=2x^2-4x+2+1$

$=2x^2-4x+3$

9. C

Let $f(x) = x^2 + px + q$

$f(-2) = (-2)^2 + p(-2) + q$

$-1 = 4 - 2p + q$

$2p - q = 5$

$4p - 2q = 10$

$4p - 2q + 5 = 15$

10. D

From the graph, we have

$a < 0$, $b < 0$, $c < 0$

For I, $ac > 0$ ✓

For II, $\dfrac{b}{c} > 0$ ✓

For III, $b + c < 0$ ✓

11. A

$\text{Cost} = \dfrac{\$8000}{1 + 60\%} = \$5000$

$\text{Profit} = \$8000(1 - 20\%) - \$5000 = \$1400$

12. B

Let A m^2 be the actual area of the park.

$$(\frac{1}{5000})^2 = \frac{8 \div 100^2}{A}$$

$$A = 20000$$

Thus, the actual area of the park is 20000 m^2.

13. A

Since, $\frac{bc}{a}$ is a non-zero constant,

Let $\frac{bc}{a}=k$ where k is a non-zero constant.

$$bc = ak$$

$$a = \frac{bc}{k}$$

$$a \propto bc$$

Thus, a varies directly as b and c.

14. D

1st pattern : 2 dots

2nd pattern : $(2 + 2^3)$ dots = 10 dots

3rd pattern : $(2 + 2^3 + 2^4)$ dots = 26 dots

4th pattern : $(2 + 2^3 + 2^4 + 2^5)$ dots = 58 dots

...

8th pattern : $(2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9)$ dots = 1018 dots

15. C

Let h cm be the rise in water level.

$$14\times16\times6+\pi(1)^2\times(h+6)=14\times16\times(h+6)$$

$$1344+\pi\times(h+6)=224(h+6)$$

$$1344=(224-\pi)(h+6)$$

$$h=\frac{1344}{224-\pi}-6$$

$$h\approx 0.0853$$

Thus, the rise in water level is 0.0853 cm.

16. D

$\angle AOB = 90^\circ$ [$\angle$ in semi-circle]

$OA = OB = 4$ cm

$AB = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ cm

Thus, the radius of the smaller circle is $2\sqrt{2}$ cm .

$$\text{The area of the shaded region} = \{\pi(4)^2 + \pi(2\sqrt{2})^2 - 2\times[\frac{\pi(2\sqrt{2})^2}{2} + \frac{\pi(4)^2}{4} - \frac{4\times 4}{2}]\}\text{ cm}^2$$

$$= [16\pi + 8\pi - 2\times(4\pi + 4\pi - 8)]\text{ cm}^2$$

$$= (24\pi - 16\pi + 16)\text{ cm}^2$$

$$= (8\pi + 16)\text{ cm}^2$$

17. B

$AX : XB = 5 : 2 = 10 : 4$

Since, the area of ΔXBZ is 4 cm^2, so the area of ΔXAZ is 10 cm^2.

Since, $BY = YC$

So, the area of ΔZYB = the area of ΔZYC

and the area of ΔAYB = the area of ΔAYC

Thus, the area of ΔAZC = the area of ΔAYC – the area of ΔZYC

= the area of ΔAYB – the area of ΔZYB

= the area of ΔABZ

= 14 cm^2

18. B

Produce AB to meet CD at a point F.

$\angle CFB = \angle CDE = x + 6^\circ$ [corr. $\angle$s, AB // ED]

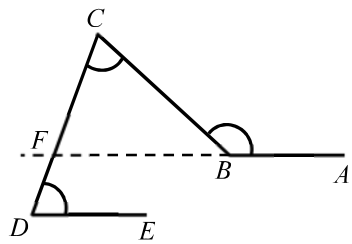
$\angle CFB + \angle BCD = \angle ABC$ [ext. $\angle$ of Δ]

$x + 6^\circ + x + 10^\circ = 4x - 98^\circ$

$2x + 16^\circ = 4x - 98^\circ$

$2x = 114^\circ$

$x = 57^\circ$



19. A

The number of folds of rotational symmetry is 24. I. ✓

Each interior angle $= \dfrac{(24-2)\times 180^\circ}{24} = 165^\circ$. II. ✓

Each exterior angle $= \dfrac{360^\circ}{24} = 15^\circ$

$\dfrac{165^\circ}{15^\circ} = 11$ times III. ×

20. D

$EC = DC = BC = CF$

$\angle ECD = \angle BCF = 60^\circ$ [properties of equilateral Δ]

$\angle BCE = 90^\circ - 60^\circ = 30^\circ$

$\angle CEF = \angle CFE$ [base. ∠s, isos. Δ]

$\angle CEF + \angle CFE + 90^\circ = 180^\circ$ [∠ sum of Δ]

$\angle CEF = 45^\circ$

$\angle CEB = \angle CBE$ [base. ∠s, isos. Δ]

$\angle CEB + \angle CBE + 30^\circ = 180^\circ$ [∠ sum of Δ]

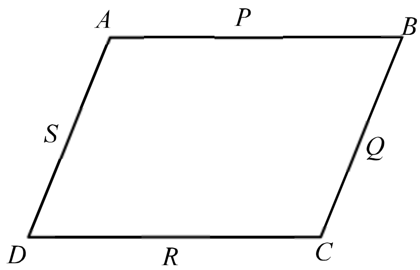
$\angle CEB = 75^\circ$

Thus, $\angle BEF = \angle CEB - \angle CEF = 75^\circ - 45^\circ = 30^\circ$

21. A

$AB = DC$, $AP = PB$, $DR = RC$

$AP = \dfrac{1}{2}AB = \dfrac{1}{2}DC = CR \Rightarrow AP = CR$ I. ✓



Similarly, $AS = CQ$

$\angle SAP = \angle QCR$

$\Delta SAP \cong \Delta QCR$ (SAS)

Thus, $SP = RQ$

Similarly, $RS = QP$

So, $PQRS$ is a parallelogram.

Thus, $\angle QPS = \angle SRQ$. [properties of //gram] II. ✓

III is correct if and only if $ABCD$ is a square or rectangle. III. ×

22. D

$\angle OAC = \angle OCA$ [base. $\angle$s, isos. Δ]

$\angle OAC + \angle OCA + \angle AOC = 180^\circ$ [$\angle$ sum of Δ]

$2\angle OCA + 108^\circ = 180^\circ$

$\angle OCA = 36^\circ$

$\angle OCD = 98^\circ - 36^\circ = 62^\circ$

$\angle ODC = \angle OCD = 62^\circ$ [base. $\angle$s, isos. Δ]

$\angle COD = 180^\circ - 62^\circ - 62^\circ = 56^\circ$ [$\angle$ sum of Δ]

$\angle EBC = \dfrac{56^\circ}{2} = 28^\circ$ [$\angle$ at centre = 2$\angle$ at Θ^{CE}]

23. C

$\cos\alpha = \dfrac{AC}{AD}$

$AC = a\cos\alpha$

$\cos\beta = \dfrac{BC}{BD}$

$BC = b\cos\beta$

$AB = AC - BC$

$= a\cos\alpha - b\cos\beta$

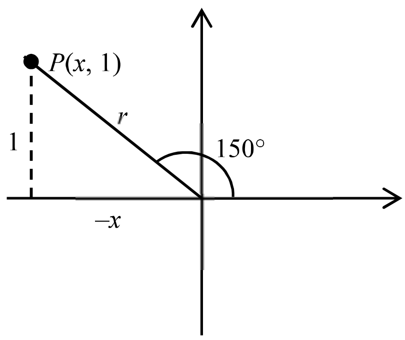
24. B

$\tan(180^\circ - 150^\circ) = \dfrac{1}{-x}$

$\tan 30^\circ = \dfrac{1}{-x}$

$\dfrac{1}{\sqrt{3}} = \dfrac{1}{-x}$

$x = -\sqrt{3}$



25. B

Let $R(k, r)$.

$$\frac{4-r}{0-k}=\frac{4-0}{0+5}$$

$$\frac{4-r}{-k}=\frac{4}{5}$$

$$20-5r=-4k$$

$$5r=20+4k$$

$$r=\frac{20+4k}{5}$$

Thus, the y-coordinate of R is $\frac{20+4k}{5}$.

26. A

The locus of P is the angle bisector of $\angle ABC$.

27. C

G_1: (4, 3), radius of $C_1=\sqrt{5}$

G_2: (–3, 4), radius of $C_2=\sqrt{8.5}$

Slope of $G_1O=\frac{3-0}{4-0}=\frac{3}{4}$

Slope of $G_2O=\frac{4-0}{=3-0}=-\frac{4}{3}$

Slope of G_1O × slope of $G_2O=\frac{3}{4}\times-\frac{4}{3}=-1$

Thus, G_1O is perpendicular to G_2O. I ✓

Area of $C_1=\pi(\sqrt{5})^2=5\pi$

Area of $C_2=\pi(\sqrt{8.5})^2=8.5\pi$ II ✗

$G_1O=\sqrt{4^2+3^2}=5$

$G_2O=\sqrt{(-3)^2+4^2}=5$ III ✓

28. B

The required probability

$=1-\frac{1}{8}-\frac{7}{8}\times\frac{1}{7}$

$=\frac{3}{4}$

29. D

Let a, b and c be the three numbers added where $a<b<c$.

Since the lower quartile is 19.

$a\le 19$

Since the median is 25.

$19\le b\le 25$

$\frac{a+b+c}{3}=29$

$a+b+c=87$

$b+c\ge 68$

$c\ge 68-b$

$43\le c\le 49$

30. A

$a=\frac{x-5+x-1+x-1+x-1+x+x+2+x+4+x+6}{8}=x+0.5$

$b=x$

$c=x-1$

$d=x+6-x+5=11$

I. $a>b$ ✓

II. $a<d$ ✗

III. $c=\frac{a+b}{2}$ ✗

31. D

$5 \times 2^8 + 17 - 48 \times 2^3$

$= 5 \times 2^8 + (2^4 + 1) - (2^4 + 2^5) \times 2^3$

$= 5 \times 2^8 + 2^4 + 1 - 2^7 - 2^8$

$= 4 \times 2^8 - 2^7 + 2^4 + 1$

$= 2^7(4 \times 2 - 1) + 2^4 + 1$

$= 2^7(7) + 2^4 + 1$

$= 2^7(2^2 + 2 + 1) + 2^4 + 1$

$= 2^9 + 2^8 + 2^7 + 2^4 + 1$

$= 1110010001_2$

32. C

For I,

a^3b^3c, $a^2b^2cd^2$, a^4bc

H.C.F. = a^2bc

L.C.M. = $a^4b^3cd^2$

I ✓

For II,

a^4b^2cd, $a^2b^2cd^2$, a^4bc

H.C.F. = a^2bc

L.C.M. = $a^4b^2cd^2$

II ✗

For III,

$a^2b^3cd^2$, $a^2b^2cd^2$, a^4bc

H.C.F. = a^2bc

L.C.M. = $a^4b^3cd^2$

III ✓

33. A

$(\log_9 x)^2 + \log_9 x^2 - 12 = \log_9 x$

$(\log_9 x)^2 + 2\log_9 x - \log_9 x - 12 = 0$

$(\log_9 x)^2 + \log_9 x - 12 = 0$

$(\log_9 x + 4)(\log_9 x - 3) = 0$

$\log_9 x = -4$ or $\log_9 x = 3$

$x = 9^{-4}$ or $x = 9^3$

$\log_3 m + \log_3 n$

$= \log_3 9^{-4} + \log_3 9^3$

$= -4\log_3 9 + 3\log_3 9$

$= -\log_3 3^2$

$= -2$

34. A

$f(x) = x^2 + bx + c$

$f(4 - 3i) = (4 - 3i)^2 + b(4 - 3i) + c$

$0 = 7 - 24i + 4b - 3bi + c$

$0 = (7 + 4b + c) - (24 + 3b)i$

So, $24 + 3b = 0$, we have $b = -8$.

$7 + 4b + c = 0$

$c = 25$

Thus, $b - c = -8 - 25 = -33$

35. A

$f(x) \le 0$

$a \le x \le b$

$f(x - a + b) \le 0$

$a \le x - a + b \le b$

$a - b \le x - a \le 0$

$2a - b \le x \le a$

For $a \le x \le b$ and $2a - b \le x \le a$, we have

$x = a$

36. B

Let r be the common ratio of the geometric sequence.

$a_2 = 4$

$a_1 r = 4$ ……(1)

$a_4 = 16$

$a_1 r^3 = 16$ ……(2)

By solving (1) and (2),

$a_1 = 2, r = 2$ or $a_1 = -2, r = -2$

For I

Consider $a_1 = -2, r = -2$

$$\frac{a_{n+1}}{a_n} = r = -2 < 0$$

I ✗

For II

$$\frac{a_{n+3}}{a_{n+2}} = \frac{a_{n+1}}{a_n} = r \Rightarrow a_n a_{n+3} = a_{n+1} a_{n+2}$$

II ✓

For III

Consider $a_1 = -2, r = -2$

$$S_{50} = \frac{(-2)[(-2)^{50} - 1]}{-2-1} = \frac{2^{51} - 2}{3}$$

$$a_{51} = (-2)(-2)^{50} = -2^{51}$$

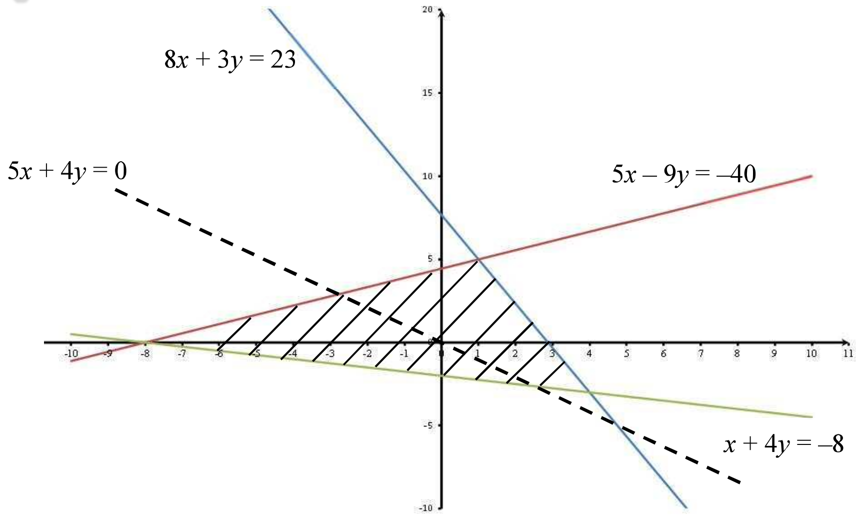
III ✗

37. A

From the graph, the maximum value attains at the point (1, 5)

$5(1) + 4(5) + c = 49$

$c = 24$



38. D

$\angle AEB = 90^\circ$ [$\angle$ in semi-circle]

$\angle AEC = \angle ABE$ [$\angle$ in alt. segment]

$\angle BAE = 180^\circ - \angle AEB - \angle ABE$ [$\angle$ sum of Δ]

$= 180^\circ - \angle ACE - \angle AEC = \angle EAC$ [$\angle$ sum of Δ]

So, AE is an angle bisector of ΔACD.

Thus, the in-centre of ΔACD lies on AE. I ✓

$\angle AEC = \angle ABE$ [$\angle$ in alt. segment]

$\angle BAE = \angle EAC$ [proved]

$\Delta ACE \sim \Delta AEB$ (AA) II ✓

$\dfrac{AC}{AE} = \dfrac{AE}{AB}$ [corr. sides, ~ Δs]

$AB \times AC = AE^2$ III ✓

39. C

Let the coordinates of the centre of C_2 be (h, k).

$$\begin{cases} x^2 + y^2 + 8x + 6y - 9 = 0 \text{ ......(1)} \\ y = -4x - 1 \text{ ......(2)} \end{cases}$$

Sub. (2) into (1),

$x^2 + (-4x-1)^2 + 8x + 6(-4x-1) - 9 = 0$

$17x^2 - 8x - 14 = 0$

The x-coordinate of the mid-point of $PQ = \dfrac{1}{2} \times \dfrac{-(-8)}{17} = \dfrac{4}{17}$

The y-coordinate of the mid-point of $PQ = -4(\dfrac{4}{17}) - 1 = -\dfrac{33}{7}$

Centre of $C_1 = (-4, -3)$

$\dfrac{-4+h}{2} = \dfrac{4}{17} \Rightarrow h = \dfrac{76}{17}$

$\dfrac{-3+k}{2} = -\dfrac{33}{17} \Rightarrow k = -\dfrac{15}{17}$

Thus, the coordinates of the centre of C_2 is $(\dfrac{76}{17}, -\dfrac{15}{17})$.

40. C

Let $PA = PB = PC = PD = AB = a$

$$EC = \sqrt{a^2 + a^2} = a\sqrt{2}$$

Height of the pyramid $PABCD = \sqrt{a^2 - (\frac{a}{2})^2 - (\frac{a}{2})^2} = \frac{a}{\sqrt{2}}$

$$PE = \sqrt{(\frac{a\sqrt{2}}{2})^2 + (a + \frac{a}{\sqrt{2}})^2} = a\sqrt{2+\sqrt{2}}$$

$$\cos\angle PEC = \frac{(a\sqrt{2+\sqrt{2}})^2 + (a\sqrt{2})^2 - a^2}{2(a\sqrt{2+\sqrt{2}})(a\sqrt{2})}$$

$$= \frac{(\sqrt{2+\sqrt{2}})^2 + (\sqrt{2})^2 - 1}{2(\sqrt{2+\sqrt{2}})(\sqrt{2})}$$

$$\angle PEC = 32.4°$$

41. A

$P(a, 0)$, $Q(-\frac{a}{4}, 0)$, $R(0, -\frac{a}{3})$

$$\frac{18-0}{0-a} \times \frac{\frac{-a}{3} - 0}{0 + \frac{a}{4}} = -1$$

$$a = -24$$

42. C

The required number

$= 3 \times 3 \times 3 \times 3 \times 2$

$= 162$

43. C

The required probability

$$= (1 \times \frac{1}{13} \times 1 \times \frac{1}{12} \times 1 \times \frac{1}{11}) + (1 \times \frac{1}{13} \times 1 \times \frac{1}{12} \times 1 \times \frac{10}{11} \times 3) + [1 \times \frac{1}{13} \times 1 \times \frac{11}{12} \times (\frac{1}{11} \times 1 + \frac{10}{11} \times \frac{10}{11}) \times 3]$$

$$= \frac{7}{33}$$

44. D

Case 1: $4 < x \le 6$

mode = 4, median = x, mean = $\frac{43+x}{7}$

For A.S.,

$$x-4=\frac{43+x}{7}-x$$

$$14x-28=43+x$$

$$13x=71$$

$$x=\frac{71}{13}$$

Case 2: $x > 6$

mode = 4, median = 6, mean = $\frac{43+x}{7}$

For A.S.,

$$6-4=\frac{43+x}{7}-6$$

$$56=43+x$$

$$x=13$$

Thus, the sum of all possible values of $x=13+\frac{71}{13}=18$

45. B

Let $\bar{x}$ be the mean of the two sets of numbers.

For $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, $\frac{(\alpha_1-\bar{x})^2+(\alpha_2-\bar{x})^2+(\alpha_3-\bar{x})^2+(\alpha_4-\bar{x})^2+(\alpha_5-\bar{x})^2}{5}=8.5$

$$(\alpha_1-\bar{x})^2+(\alpha_2-\bar{x})^2+(\alpha_3-\bar{x})^2+(\alpha_4-\bar{x})^2+(\alpha_5-\bar{x})^2=42.5$$

For $\{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$, $\frac{(\beta_1-\bar{x})^2+(\beta_2-\bar{x})^2+(\beta_3-\bar{x})^2+(\beta_4-\bar{x})^2+(\beta_5-\bar{x})^2+(\beta_6-\bar{x})^2}{6}=5.2$

$$(\beta_1-\bar{x})^2+(\beta_2-\bar{x})^2+(\beta_3-\bar{x})^2+(\beta_4-\bar{x})^2+(\beta_5-\bar{x})^2+(\beta_6-\bar{x})^2=31.2$$

The required standard deviation $=\sqrt{\frac{42.5+31.2}{5+6}}=2.59$